T3D.6 : Quantum and Secure Communications

Variational Quantum Demodulation for Coherent Optical Multi-Dimensional QAM

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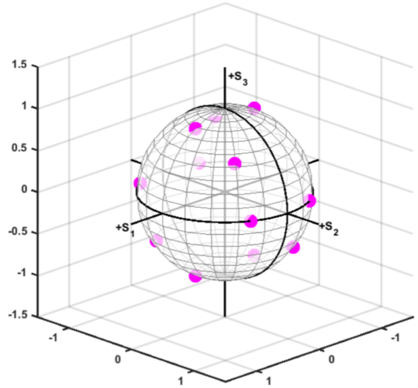
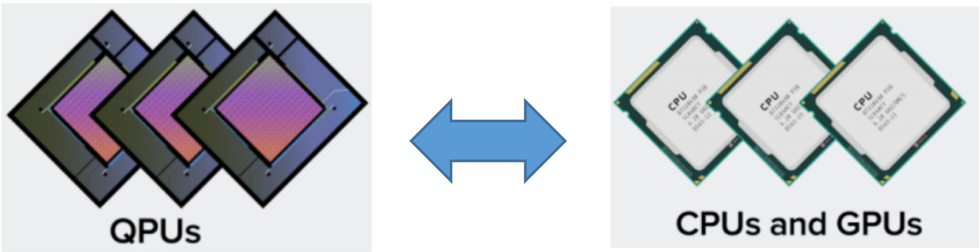
Outline

- High-dimensional modulation (HDM)
- Quantum technology trend
  - Quantum approximate optimization algorithm (QAOA)
- QAOA hybrid quantum-classical demodulation
- Simulation and real quantum processor results

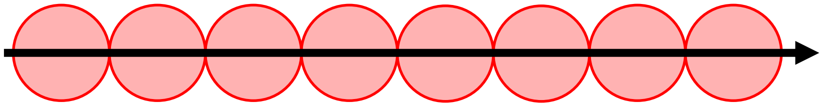




High-Dimensional Modulation (HDM)

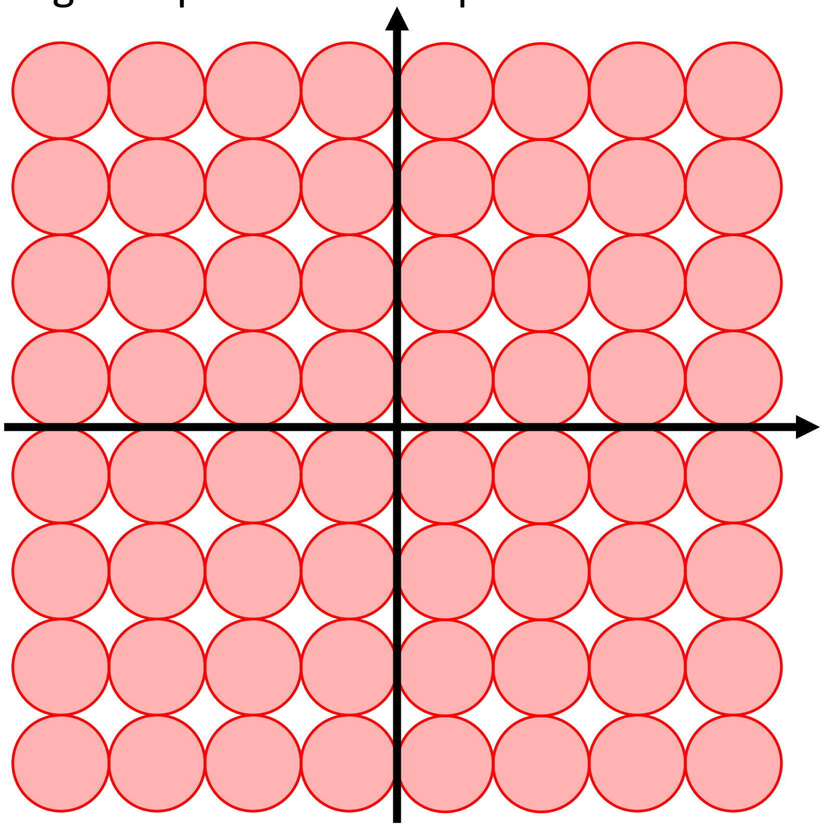
Regular lattice in 1-D:

Regular pulse-amplitude modulation (PAM)

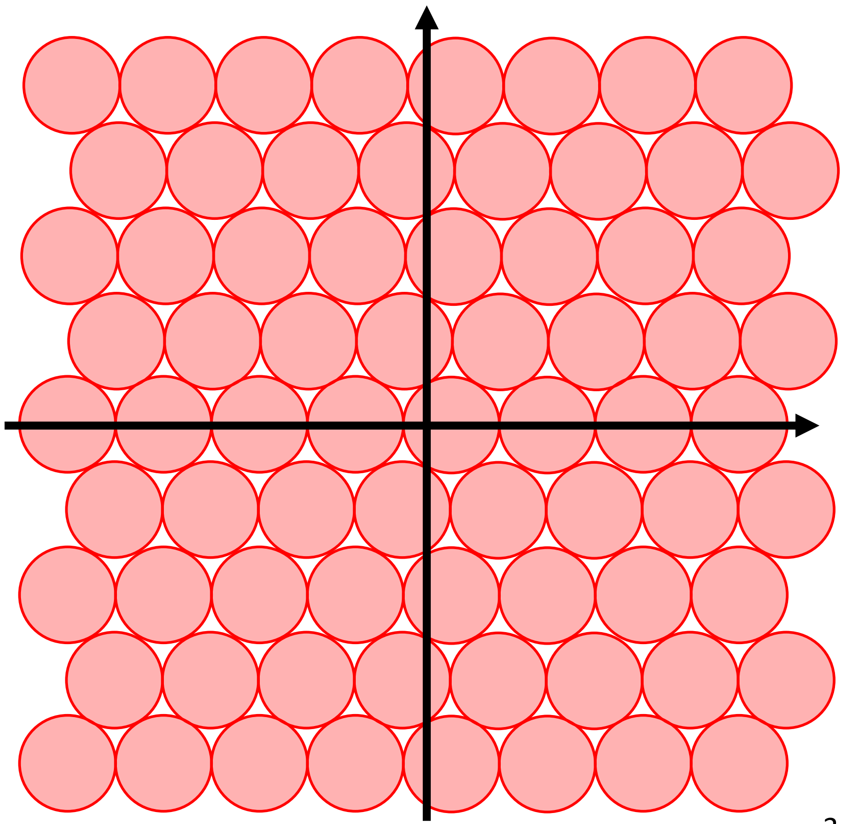


Lattice in 2-D:

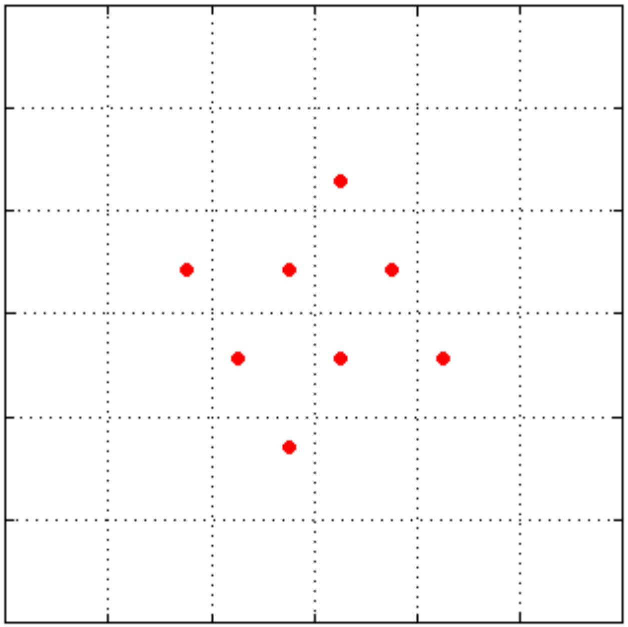
Regular quadrature-amplitude modulation (QAM)



Optimally packed lattice in 2-D:
Hexagonal lattice: max 0.8dB gain



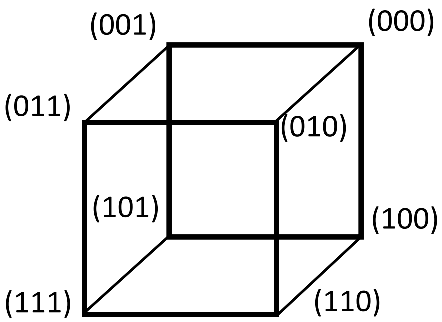
Spherical Cutting of Lattices

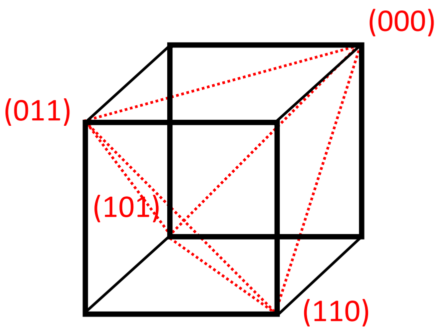


- Start with lattice in N-dimensions
- Select 2^p neighboring points
- Optimize selection
- Remove mean
- Labeling?

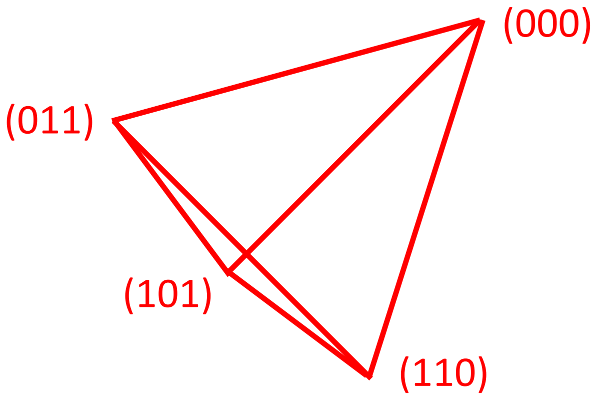
Block-Coded HDM

- Selecting a good subset of non-adjacent points from a Gray-coded hypercube





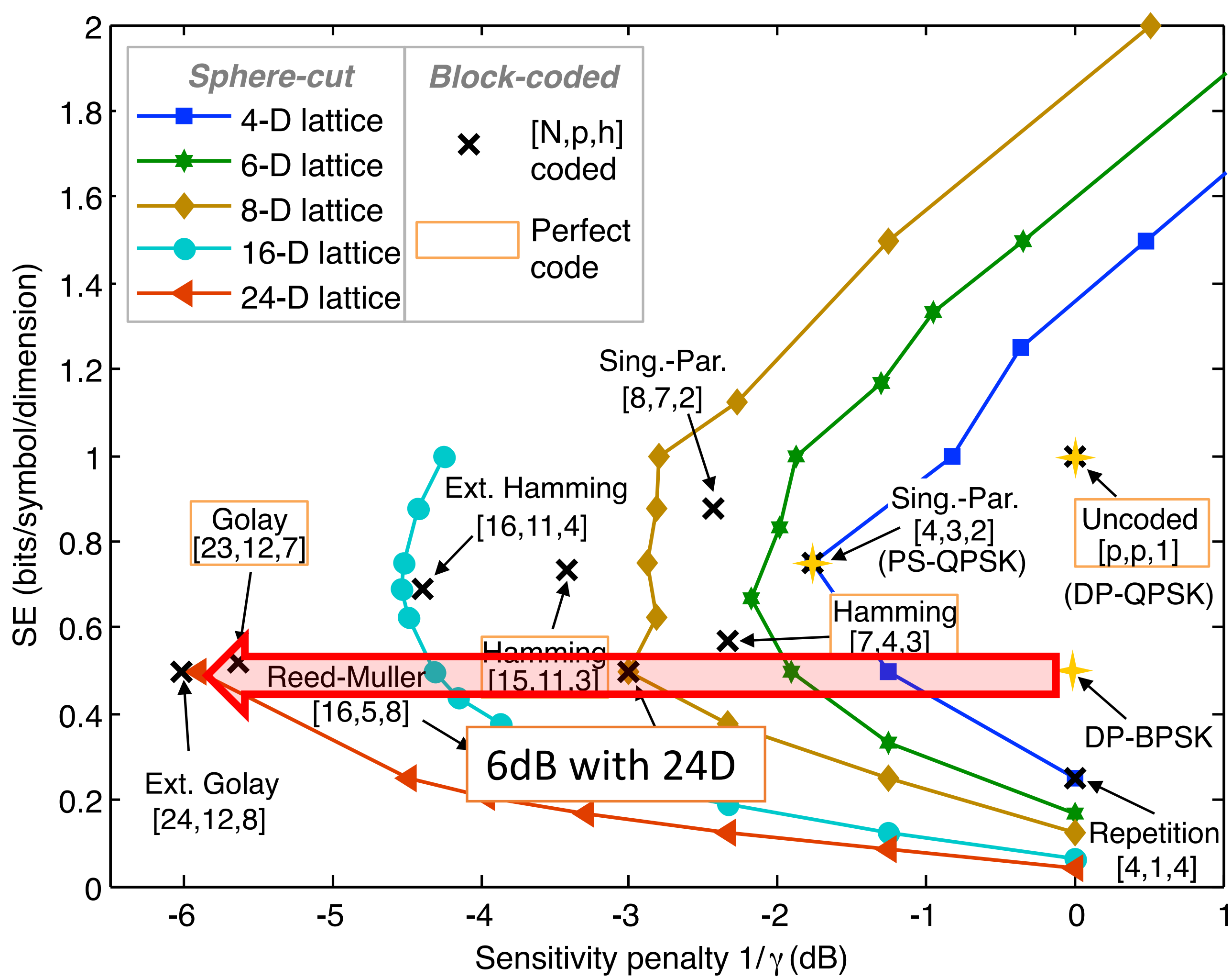
- Codewords define the set of points that are selected; e.g., [3,2,2] single-parity-check (SPC) code



- Labeling optimization is not required

Packing Gain of High-Dimensional Modulation

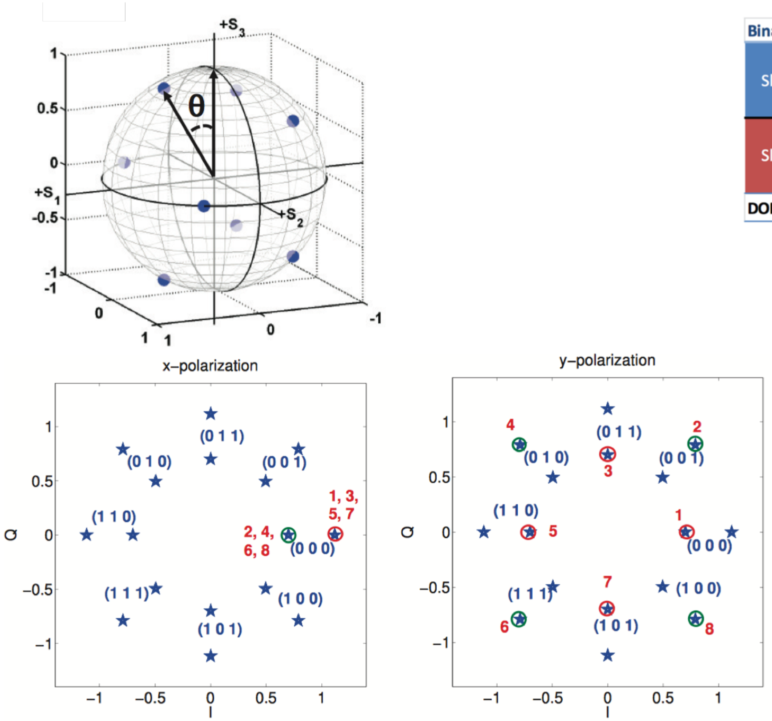
- Increasing dimensionality provides higher power efficiency: $\gamma = d_{min}^2/(4\varepsilon_b)$



Nonlinearity-Mitigating HDM

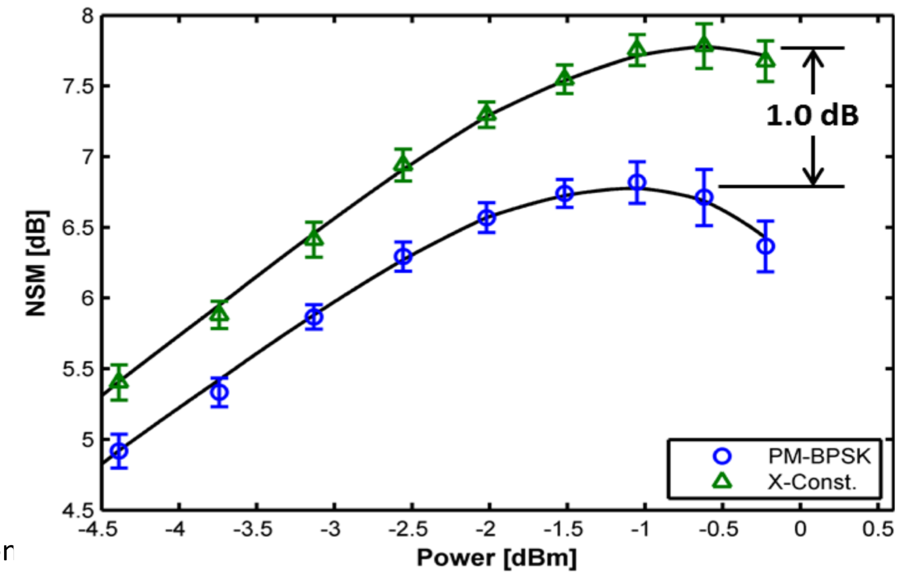
- Some HDM can further mitigate nonlinear interference (NLI)
  - 4-dimensional constant-modulus modulation (4D-2A8PSK) [Kojima et al., ECOC14]
  - 8-dimensional Grassmann modulation [Koike-Akino et al., SPPCom15]
  - 8-dimensional X-constellation (8D-X) [Shiner et al., OpEx14]

Constant modulus



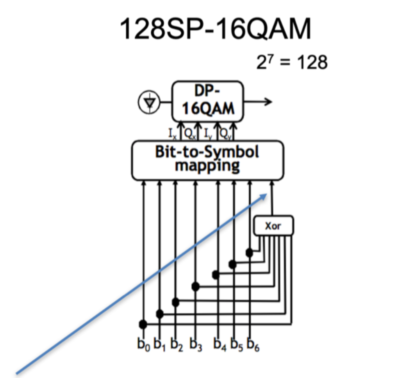
Zero degree of polarization (DOP)

Binary Value		0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
Slot A	x-pol	-1-i	-1-i	-1-i	-1-i	-1+i	-1+i	-1+i	-1+i	1-i	1-i	1-i	1-i	1+i	1+i	1+i	1+i
	y-pol	1+i	-1+i	1-i	-1-i	-1-i	1-i	-1+i	1+i	-1-i	1-i	-1+i	1+i	1+i	-1+i	1-i	-1-i
	$S_A=(S_1,S_2,S_3)$	(0,-4,0)	(0,0,-4)	(0,0,4)	(0,4,0)	(0,0,4)	(0,-4,0)	(0,4,0)	(0,0,-4)	(0,0,-4)	(0,4,0)	(0, -4, 0)	(0,0,4)	(0,4,0)	(0,0,4)	(0,0,-4)	(0,-4, 0)
Slot B	x-pol	1+i	-1+i	1-i	-1-i	1+i	-1+i	1-i	-1-i	1+i	-1+i	1-i	-1-i	1+i	-1+i	1-i	-1-i
	y-pol	1+i	-1-i	-1-i	1+i	1-i	-1+i	-1+i	1-i	-1+i	1-i	1-i	-1+i	-1-i	1+i	1+i	-1-i
	$S_B=(S_1,S_2,S_3)$	(0,4,0)	(0,0,4)	(0,0,-4)	(0,-4,0)	(0,0,-4)	(0,4,0)	(0,-4,0)	(0,0,4)	(0,0,4)	(0,-4,0)	(0,4,0)	(0,0,-4)	(0,-4,0)	(0,0,-4)	(0,0,4)	(0,4,0)
DOP	$\|S_A+S_B\|$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0



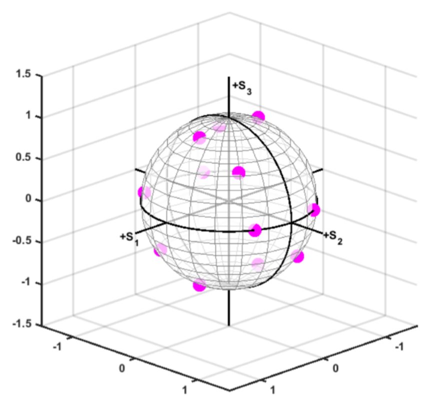
HDM Demodulation

- In exchange of higher packing gain, HDM can increase the demodulation complexity for higher dimensions
- Log-likelihood ratio (LLR) calculation is not straightforward in general to feed into soft-decision decoders
- Simplified LLR calculation based on min-sum belief propagation over algebraic constrains
- We propose quantum processing unit (**QPU**)-based demodulator



$b_7 = b_0 \oplus b_1 \oplus b_2 \oplus b_3 \oplus b_4 \oplus b_5 \oplus b_6$

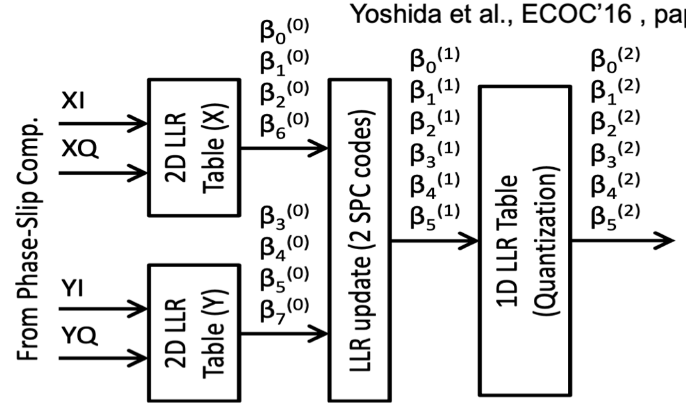
Renaudier et al., ECOC'12, We.1.C.5



$B[5] = B[0] \oplus B[1] \oplus B[2]$,
$B[6] = B[2] \oplus B[3] \oplus B[4]$.
$B[7] = \overline{B[6]}$.

Kojima et al. JLT 17

Yoshida et al., ECOC'16 , paper Th.2.P2.SC3.27



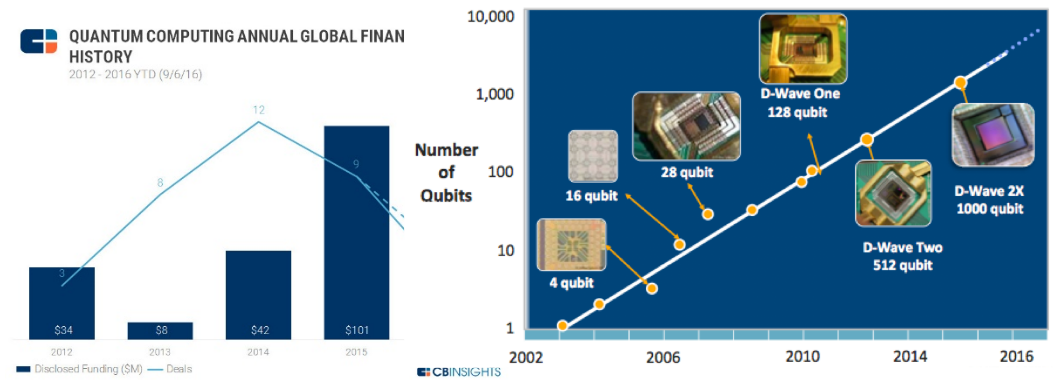
$$\alpha_{i,6} = \prod_{k=s(6),k\neq i} \mathrm{sgn}(\beta_k^{(0)}) \min_{k=s(6),k\neq i} |\gamma_k \beta_k^{(0)}|$$

$$\alpha_{i,7} = -\prod_{k=s(7),k\neq i} \mathrm{sgn}(\beta_k^{(0)}) \min_{k=s(7),k\neq i} |\gamma_k \beta_k^{(0)}|.$$

$$\beta_i^{(1)} = \beta_i^{(0)} + \varepsilon(\alpha_{i,6} + \alpha_{i,7})$$

Quantum Computing

- Morgan Stanley: Quantum tech can drive **4^{th} industrial revolution**
- Escalating government funds: National Quantum Initiative **$1.2B**
- Quantum chip providers: **IBM**, **Google**, **Microsoft**, **Honeywell**, Intel, Nokia, AirBus, IONQ, rigetti



Free libraries to evaluate quantum computing on realistic simulators or real devices

QISKit
Quantum Information Software Kit

PYQUIL

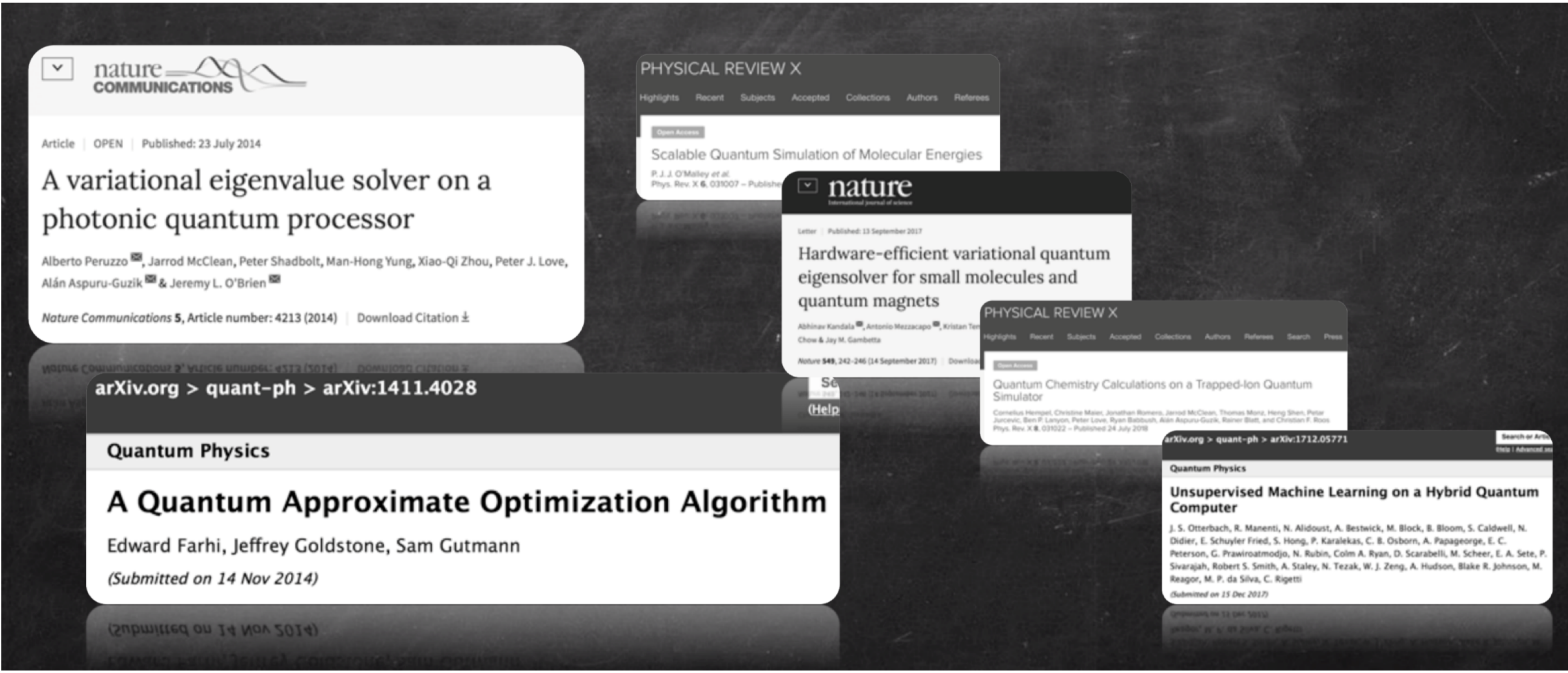
Cirq

QuTiP
Quantum Toolbox in Python

PROJECTQ

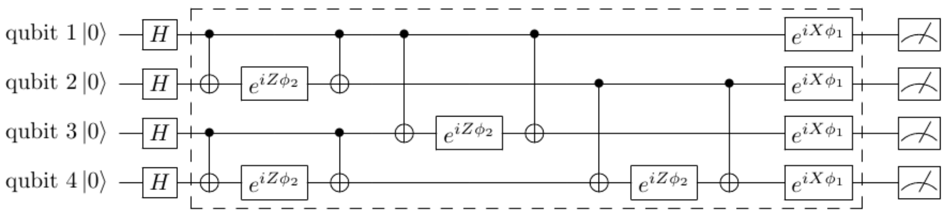
Post-2014 Trend: Variational Quantum Principle

- Hybrid use of quantum measurement and classical optimization
  - VQE: Variational Quantum Eigensolver (2014)
  - QAOA: Quantum Approximate Optimization Algorithm (2014)
  - VQF: Variational Quantum Factoring (2018)

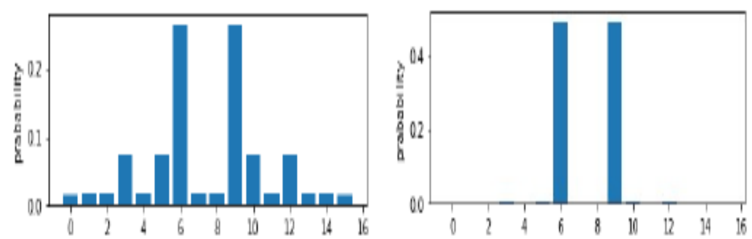


Variational Quantum Algorithms for NISQ

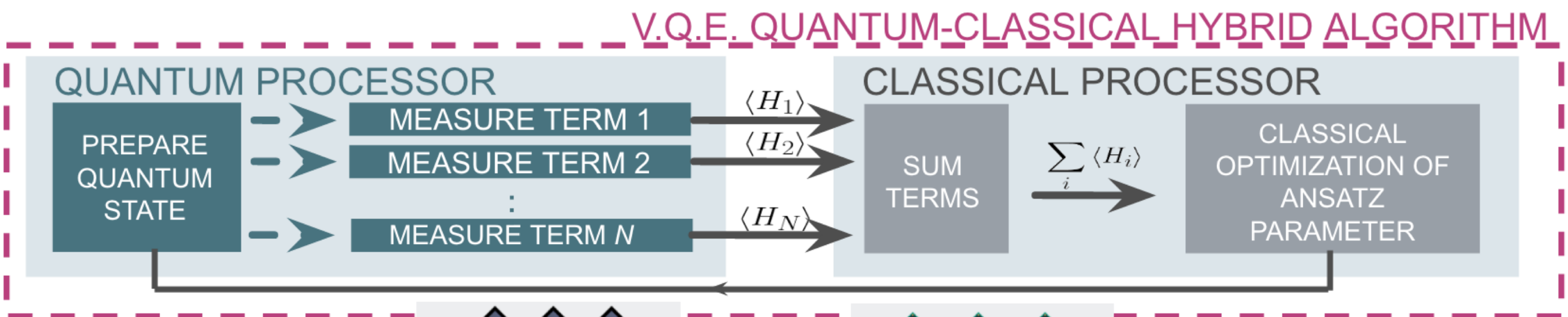
- Current quantum processors are noisy and coherence-limited: quantum gates are imperfect

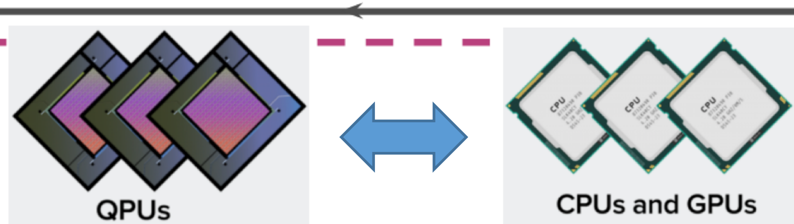


Real Ideal



- For noisy intermediate-scale quantum (**NISQ**) devices, variational hybrid quantum-classical algorithms may be a viable driver for quantum supremacy due to shallow gates and noise resilience





QAOA: Quantum Approximate Optimization Alg.

- Alternating cost Hamiltonian and mixer Hamiltonian like annealing
- Convergence theorem to eigenstate for infinite-level QAOA
  - Infinite Suzuki-Trotter decomposition with adiabatic annealing
- Classical optimization of variational angle parameters given quantum measurement

$$\lim_{p\to\infty} F_p^\star = \max_{\mathbf{z}} C(\mathbf{z})$$

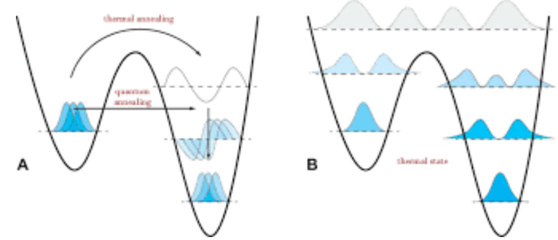
- Theoretical analysis showed better accuracy than classical counterparts; e.g. MaxCut, MaxSat, MaxClique

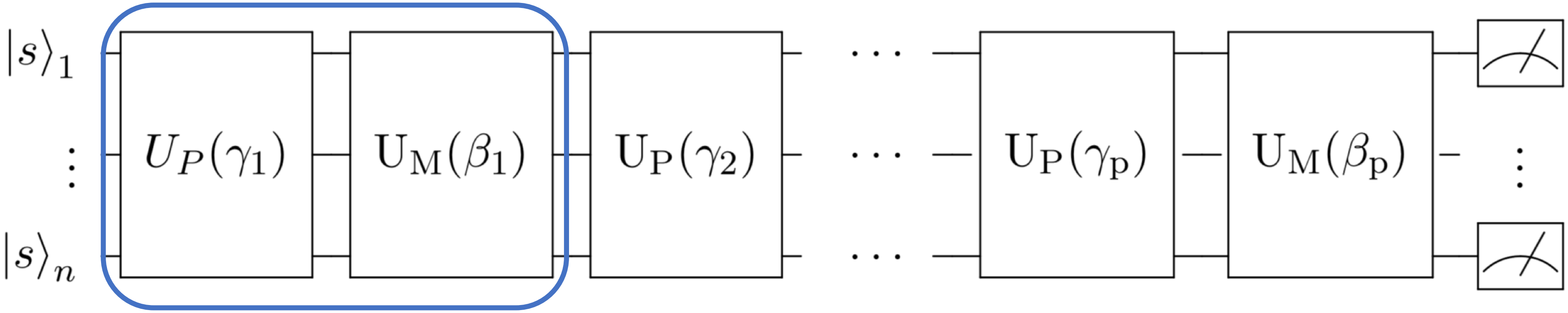
A Quantum Approximate Optimization Algorithm

Edward Farhi and Jeffrey Goldstone

Center for Theoretical Physics

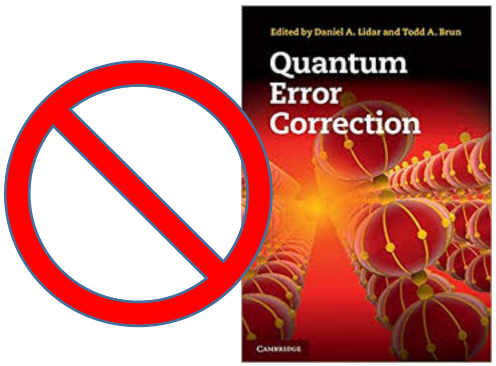
Massachusetts Institute of Technology

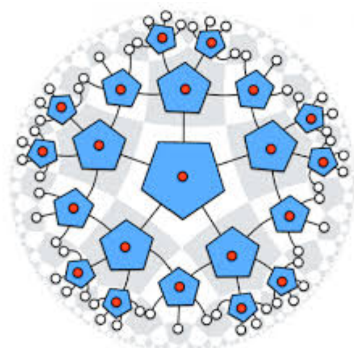


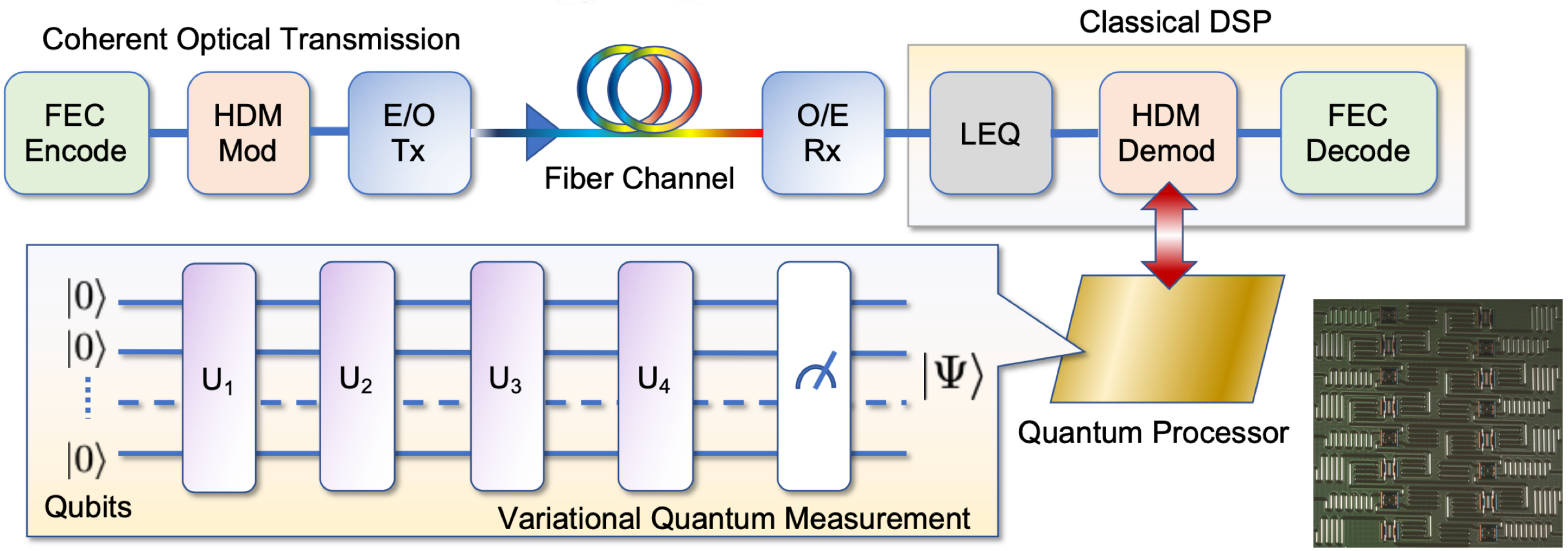


Quantum Application to Optical DSP

- This talk is **not** "Quantum Error Correction Codes (**QECC**)" nor "Quantum Communications"
- We want to offload classical demodulation computation on CPU towards QPU







Classical Channel Decoding

- Hamming codes, Reed-Muller codes, Golay codes, convolutional codes, turbo codes, low-density parity-check (LDPC) codes, polar codes, …
- Suppose linear binary codes with generator matrix $\mathbf{G} \in \mathbb{F}_2^{k \times n}$

Redundancy: Parity

[010111001010111] → [010111001010111 00100101110111011]

$\mathbf{u} \in \mathbb{F}_2^k$

$$\mathbf{x} = \mathbf{u}\mathbf{G}$$

$\mathbf{x} \in \mathbb{F}_2^n$

- Communication channel exhibits noise

$$\mathbf{y} = \mathbf{x} + \mathbf{w}$$

$\mathbf{w} \in \mathbb{R}^n$

- Maximum-likelihood (ML) decoding for symmetric channels:

$$\arg\min_{\mathbf{u}} d_{\mathrm{H}}(\mathbf{y}|\mathbf{x}) = \arg\max_{\mathbf{u}} \sum_{\nu=1}^{n} (1 - 2y_\nu)(1 - 2x_\nu)$$

NP-hard 2^k search for maximum correlation

QAOA Channel Decoding

- Convert ML decoding problem into Ising Hamiltonian model [Koike-Akino ISIT19]

$$\arg\min_{\mathbf{u}} d_{\mathrm{H}}(\mathbf{y}|\mathbf{x}) = \arg\max_{\mathbf{u}} \sum_{\nu=1}^{n} (1-2y_\nu)(1-2x_\nu)$$

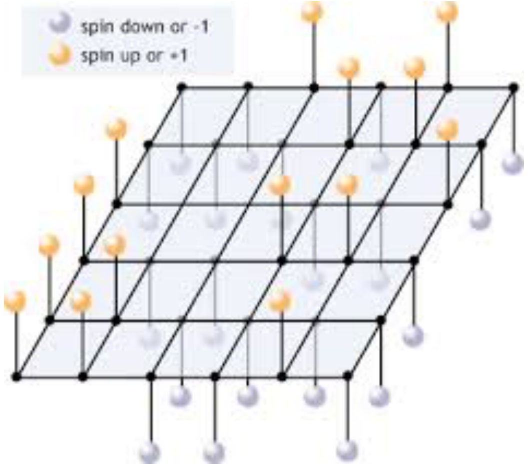
$$\mathbf{x} = \mathbf{uG}$$

k-bit search: 2^k

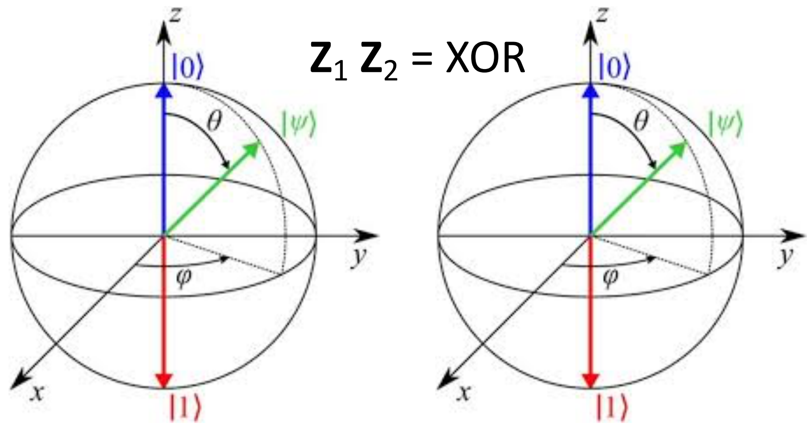
k-qubit parallel operation

$$C = \sum_{\nu=1}^{n} C_\nu = \sum_{\nu=1}^{n} (1-2y_\nu) \prod_{\kappa \in \mathcal{I}_\nu^{\mathrm{c}}} \mathbf{Z}_\kappa$$

Pauli-Z



$\mathbf{Z}_1\, \mathbf{Z}_2$ = XOR

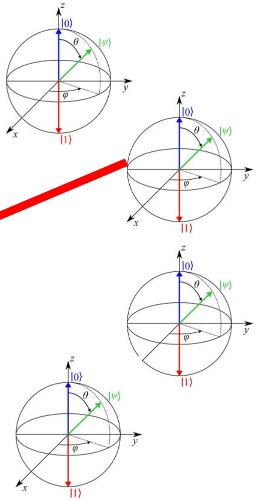


Example: Hamming Code Hamiltonian

- [7, 4]-Hamming code with minimum distance 3, corrects 1 bit error
- Generator matrix:

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Degree: $[1, 1, 1, 1, 3, 3, 3]$



$$C = r_1 \mathbf{Z}_1 + r_2 \mathbf{Z}_2 + r_3 \mathbf{Z}_3 + r_4 \mathbf{Z}_4$$

$$+ r_5 \mathbf{Z}_1 \mathbf{Z}_2 \mathbf{Z}_4 + r_6 \mathbf{Z}_1 \mathbf{Z}_3 \mathbf{Z}_4 + r_7 \mathbf{Z}_2 \mathbf{Z}_3 \mathbf{Z}_4$$

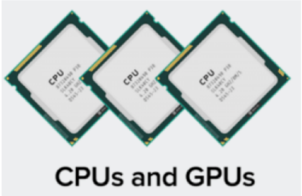
Degree-1

Degree-3

c.f.) MaxCut Hamiltonian is regular degree-2

QAOA Channel Decoding

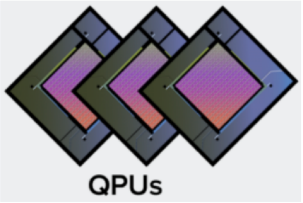
- CPU:
  - Given generator matrix G and received signal y
  - Construct cost Hamiltonian with variational angles
  - Quantum shots on QPU to obtain quasi-ML decision
  - Re-optimize angles if necessary and re-shot

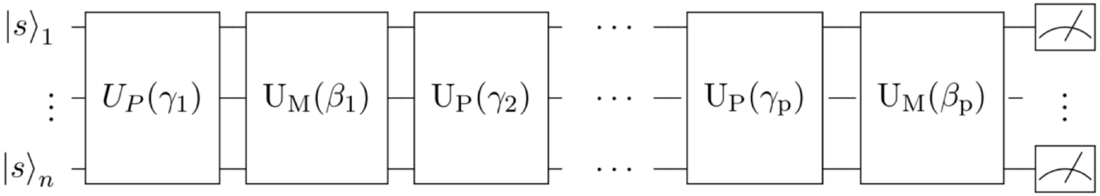


$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$C = r_1\mathbf{Z}_1 + r_2\mathbf{Z}_2 + r_3\mathbf{Z}_3 + r_4\mathbf{Z}_4 + r_5\mathbf{Z}_1\mathbf{Z}_2\mathbf{Z}_4 + r_6\mathbf{Z}_1\mathbf{Z}_3\mathbf{Z}_4 + r_7\mathbf{Z}_2\mathbf{Z}_3\mathbf{Z}_4$$

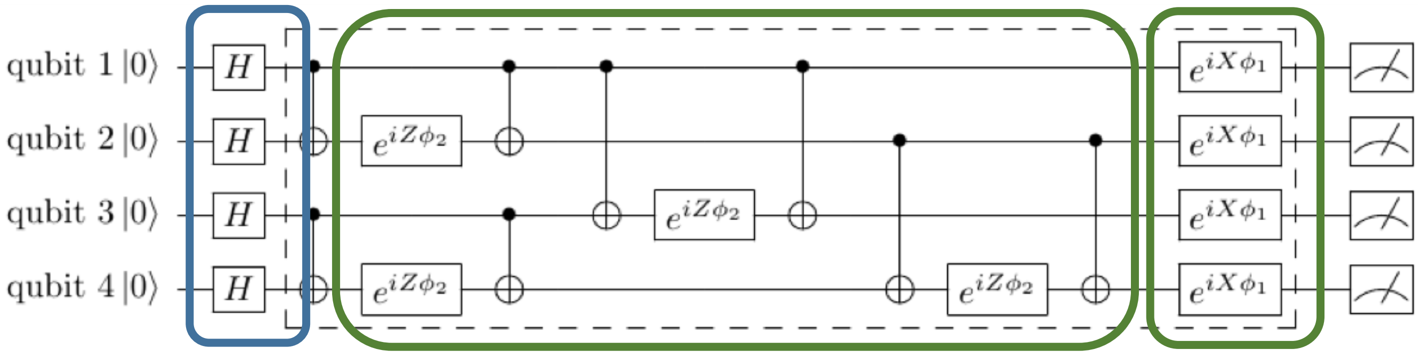
- QPU: QAOA
  - Initialize quantum state: |+> with Hadamard gates
  - Apply gamma angle rotation with cost Hamiltonian **C**
  - Apply beta angle rotation with mixer Hamiltonian **B**
  - Cascade *p*-times for level-*p* QAOA
  - Measure





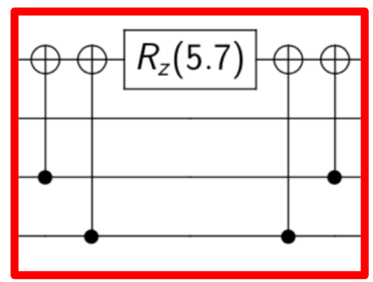
Quantum Circuits for QAOA Decoding

- State preparation: Hadamard *H*
- Mixer Hamiltonian operation: $\exp(\jmath\beta\mathbf{B})$
- Cost Hamiltonian operation: $\exp(\jmath\gamma\mathbf{C})$

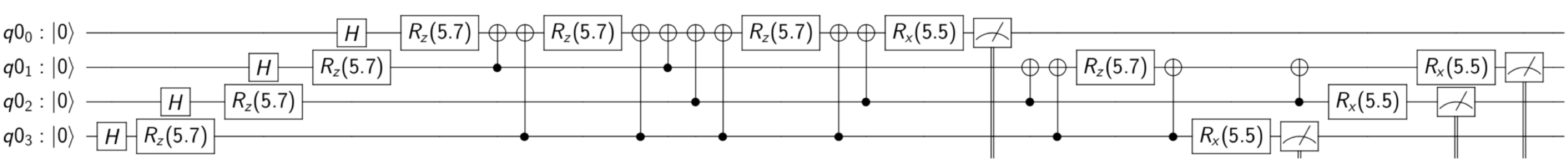


x *p*

Degree-*d* XOR: 2(*d*-1) CNOT

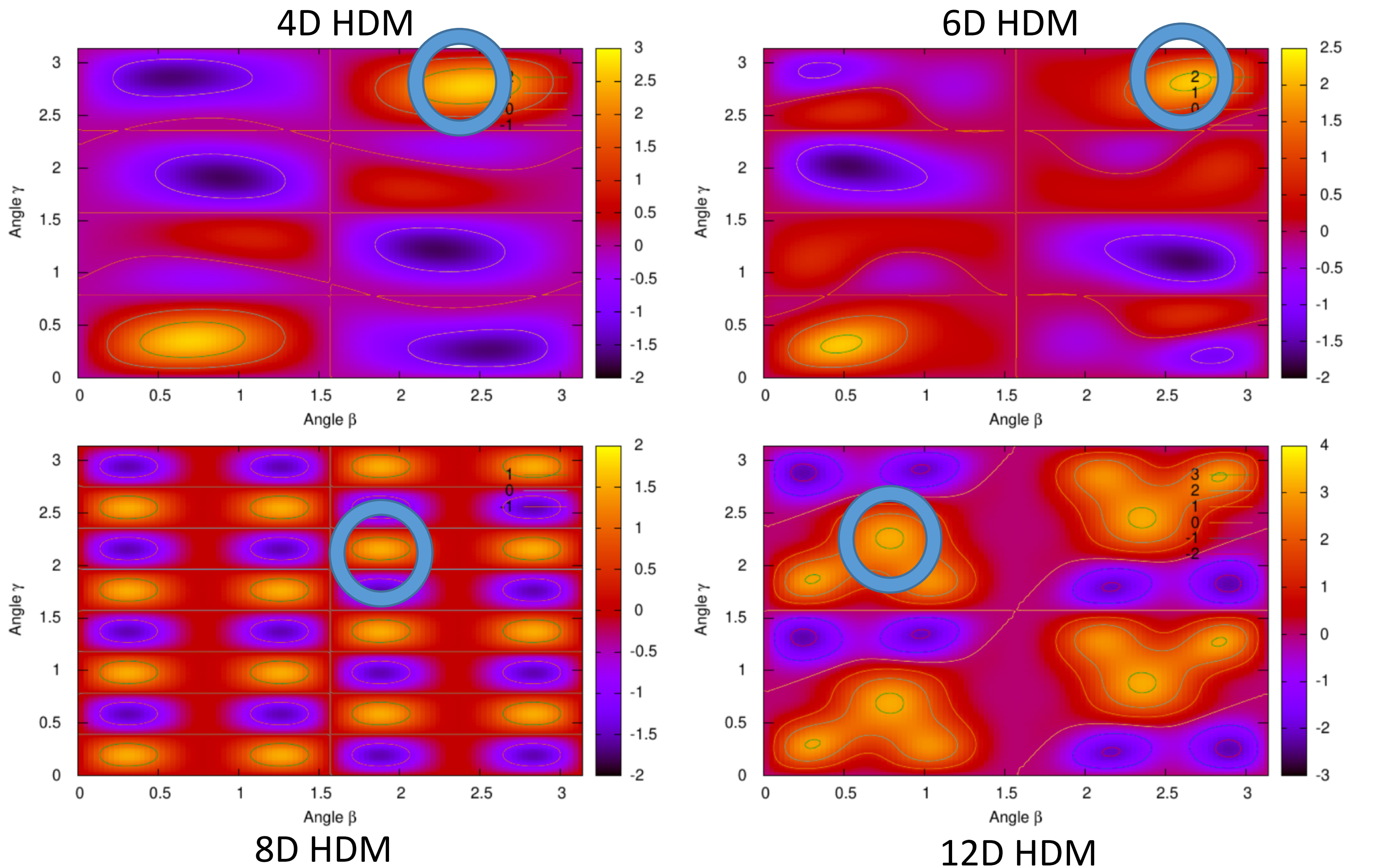


Hamming code QAOA decoder



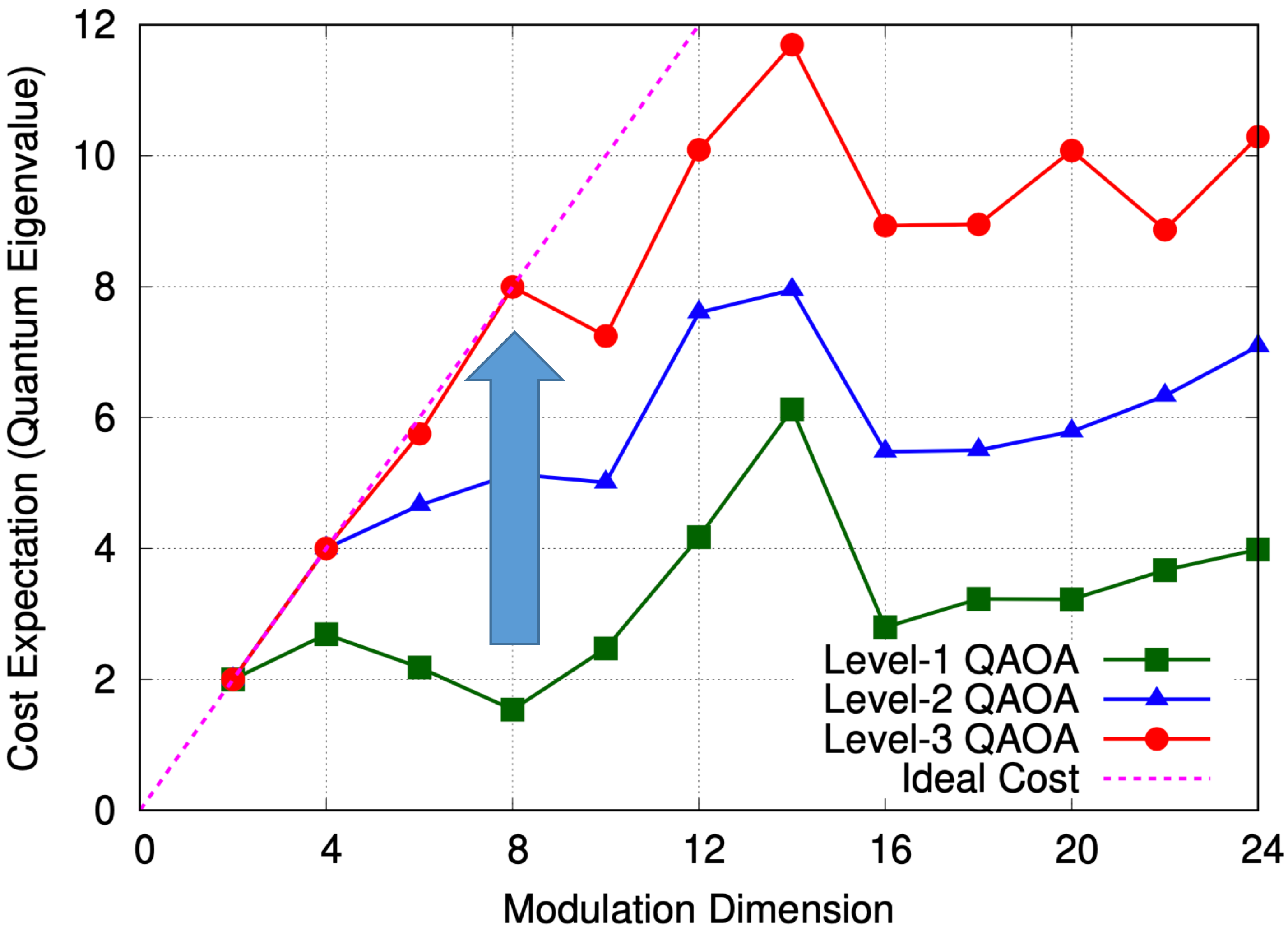
Variational Angle Optimization for HDM

- We can obtain optimal variational angles through VQE
- Landscape of cost expectation (quantum eigenvalue)



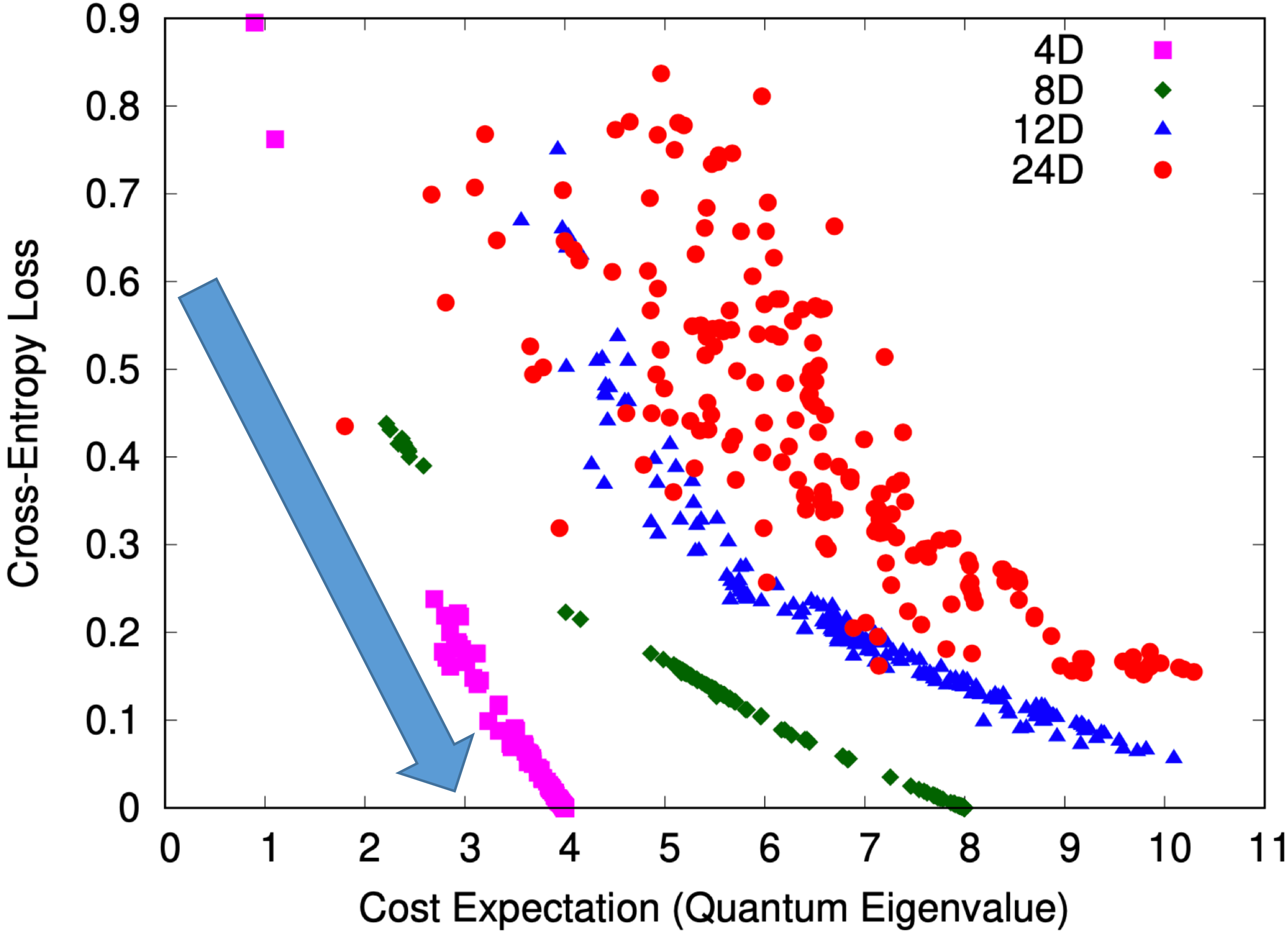
High-Level QAOA Gain

- Using higher-level QAOA, quantum eigenvalue can approach ideal quantum eigenvalue



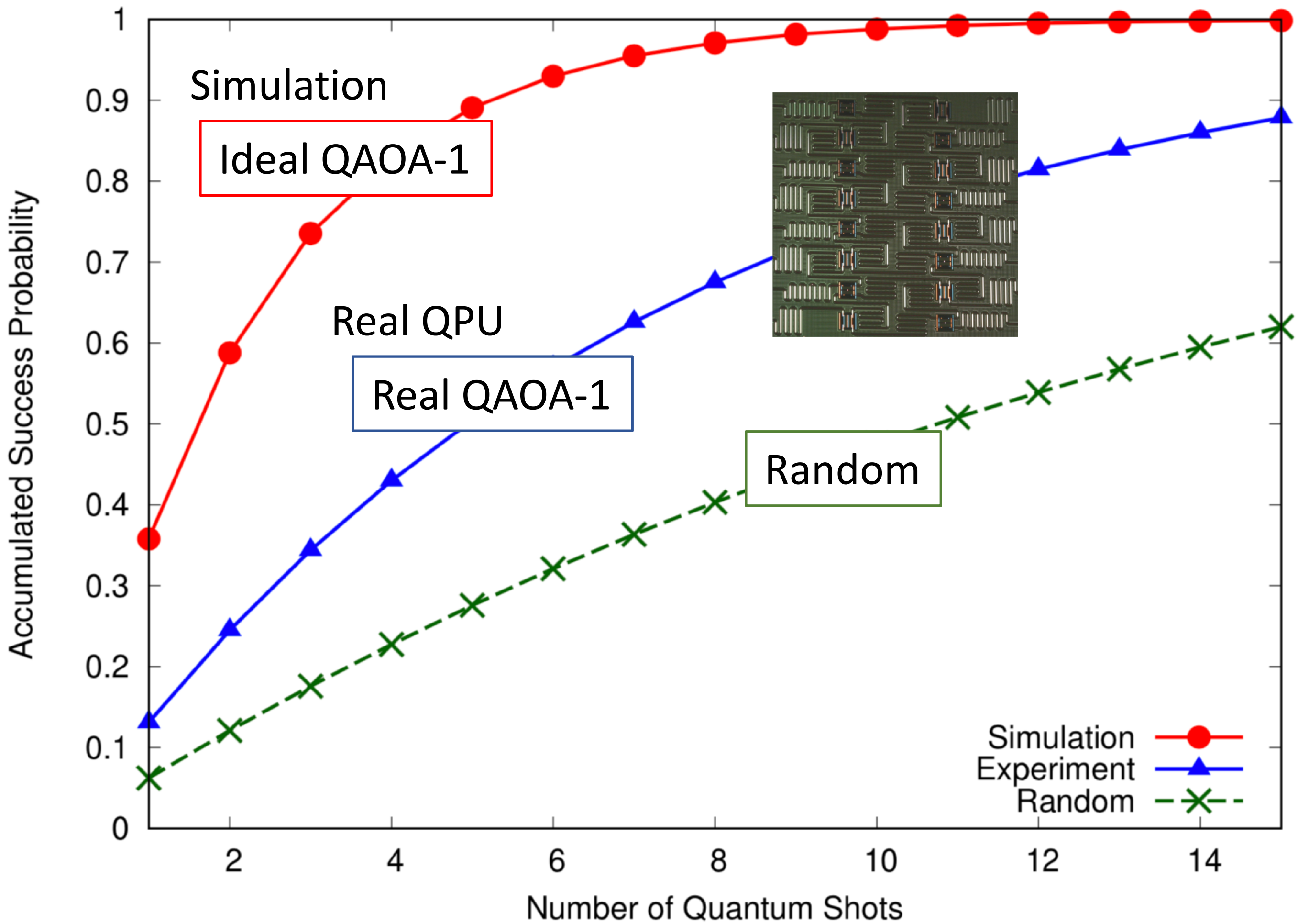
Quantum Eigenvalue and Cross Entropy

- Higher cost expectation (quantum eigenvalue) leads to smaller cross entropy loss, i.e., higher generalized mutual information (GMI)



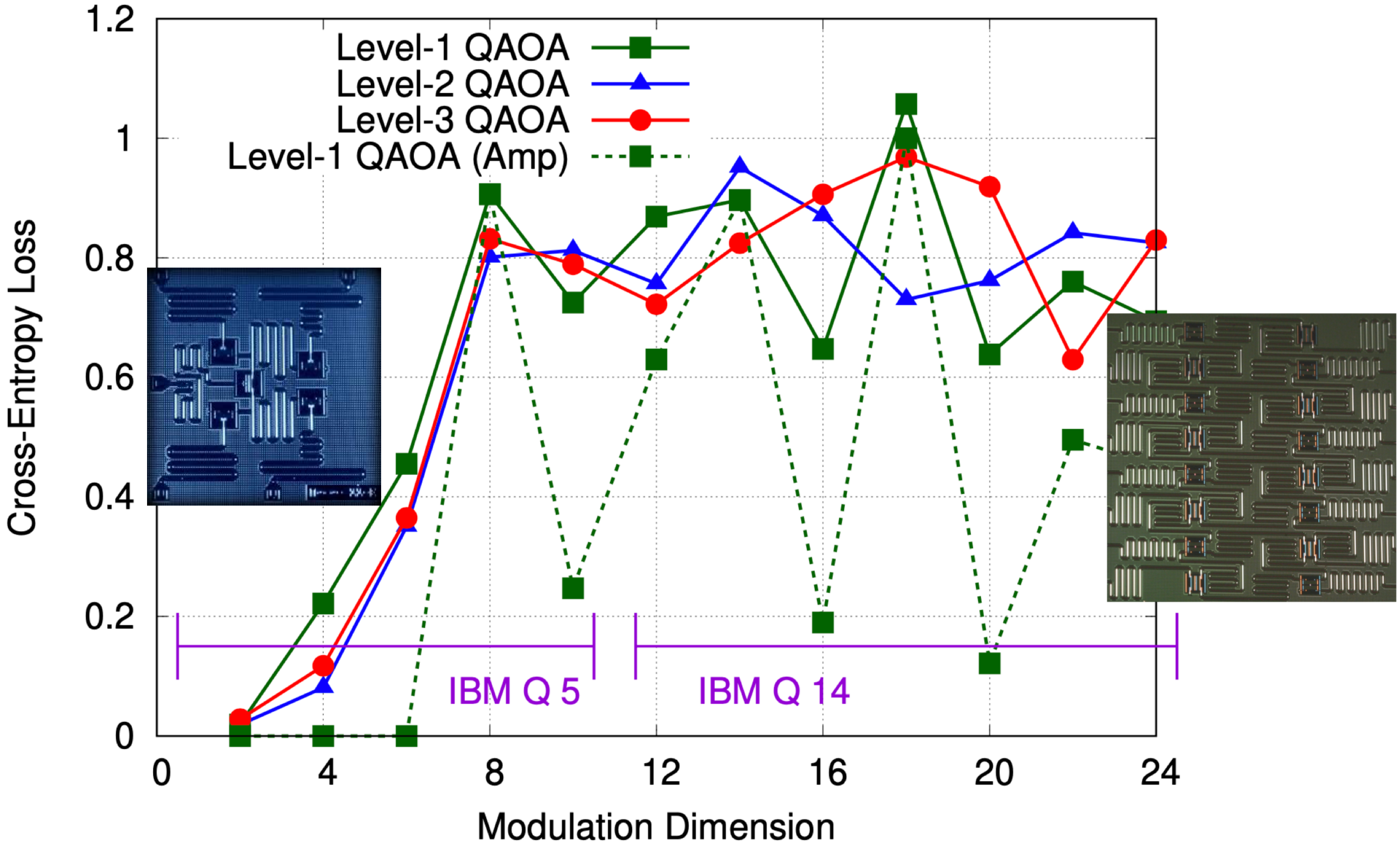
Performance on Simulated CPU and Real QPU

- ML decision success probability can be improved by taking multiple measurements of QPU shots



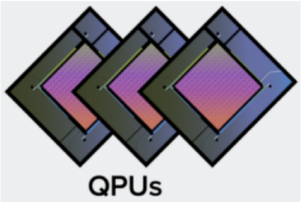
Real QPU Performance of HDM Demodulation

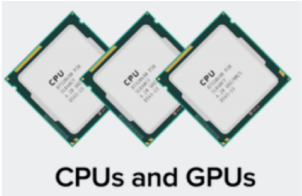
- Due to quantum errors, higher-level QAOA had no significant gain in real QPUs
- To optimize angles under decoherence scenarios
- Wavefunction amplification helps improving the performance



Conclusions

- We investigated HDM demodulation using quantum processor
- We introduced variational hybrid quantum-classical algorithms for HDM demodulation
  - QAOA demodulation was demonstrated to approach ML performance
  - Higher-level QAOA achieves better performance
- We also showed feasibility on real quantum processor
  - Quantum decoherence degraded performance
  - Parameter finetuning is required on real QPU
- To the best of our knowledge, this is the first proof-of-concept paper investigating variational quantum algorithms for HDM demodulation
  - Further rigorous analysis and improvement to follow
  - Developing quantum-ready DSP algorithms may be important for future quantum era





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