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OECC/PSC 2019
24th OptoElectronics and Communications Conference/
International Conference on Photonics in Switching and Computing 2019
Date: July 7–11, 2019 Venue: Fukuoka International Congress Center, Fukuoka

TuB3-1
Coding and Equalization

MITSUBISHI ELECTRIC RESEARCH LABORATORIES
Cambridge, Massachusetts

Hardware-Efficient Quantized Polar Decoding with Optimized Lookup Table

(Invited)

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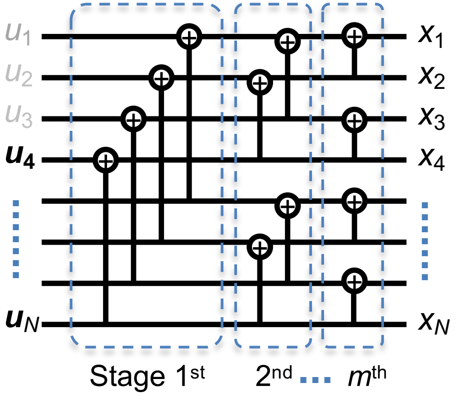
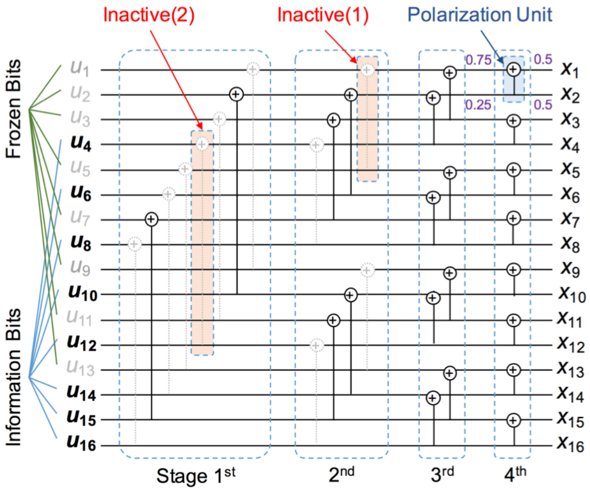
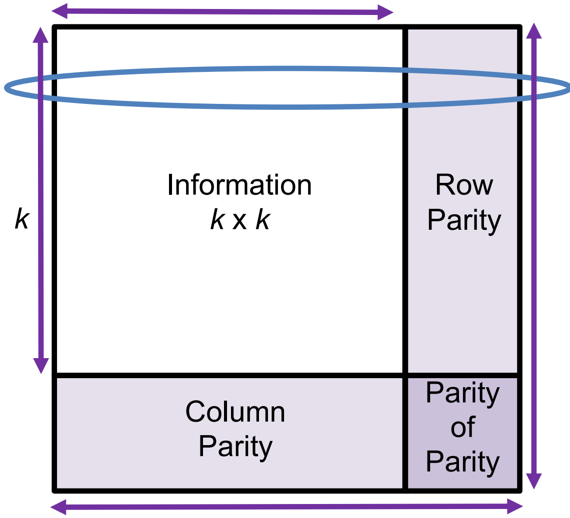
Outline

- Polar coding background
  - Successive cancellation list (SCL) decoding + cyclic-redundancy check (CRC)
  - Polar codes vs. low-density parity-check (LDPC) codes
- Polar design for bit-interleaved coded modulation (BICM)
  - Interleaver design for quadrature-amplitude modulation (QAM)
  - Non-uniform shaped QAM
- Polar-based turbo product codes (TPC)
  - Highly-parallel and pipelining processing
  - SCL-based soft-in soft-output decoding
- Irregular polar coding
  - Pruning polarization units
  - Complexity & latency reduction
- Quantized polar decoding
  - Hardware-friendly operation
  - Look-up decoding optimization
- Summary

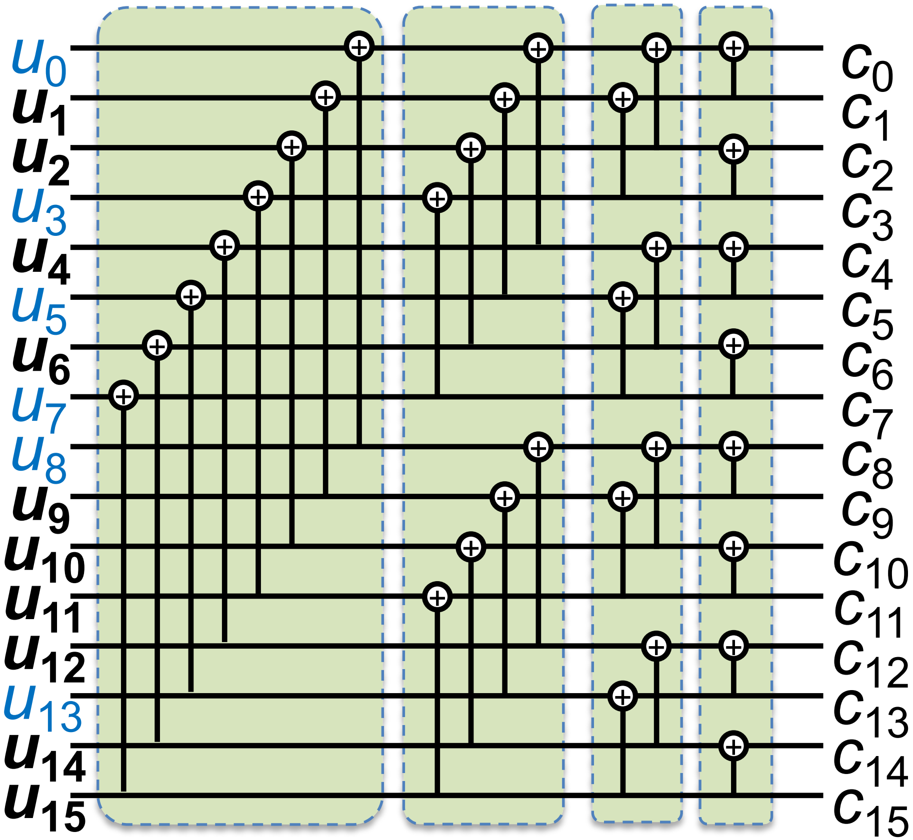






Why Polar Codes?

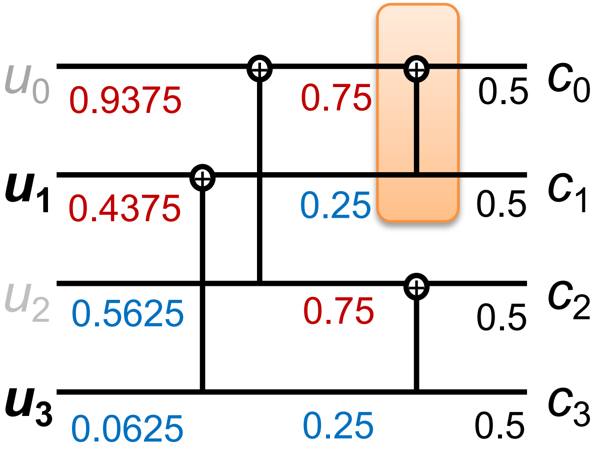
- Arikan proposed in 2008:
  - Capacity-achieving code in arbitrary discrete memoryless channels with proof
  - Low-complexity encoding and decoding; Cooley-Tukey-like butterfly architecture
  - Flexible in code rates with frozen bit selection
  - 5G standard

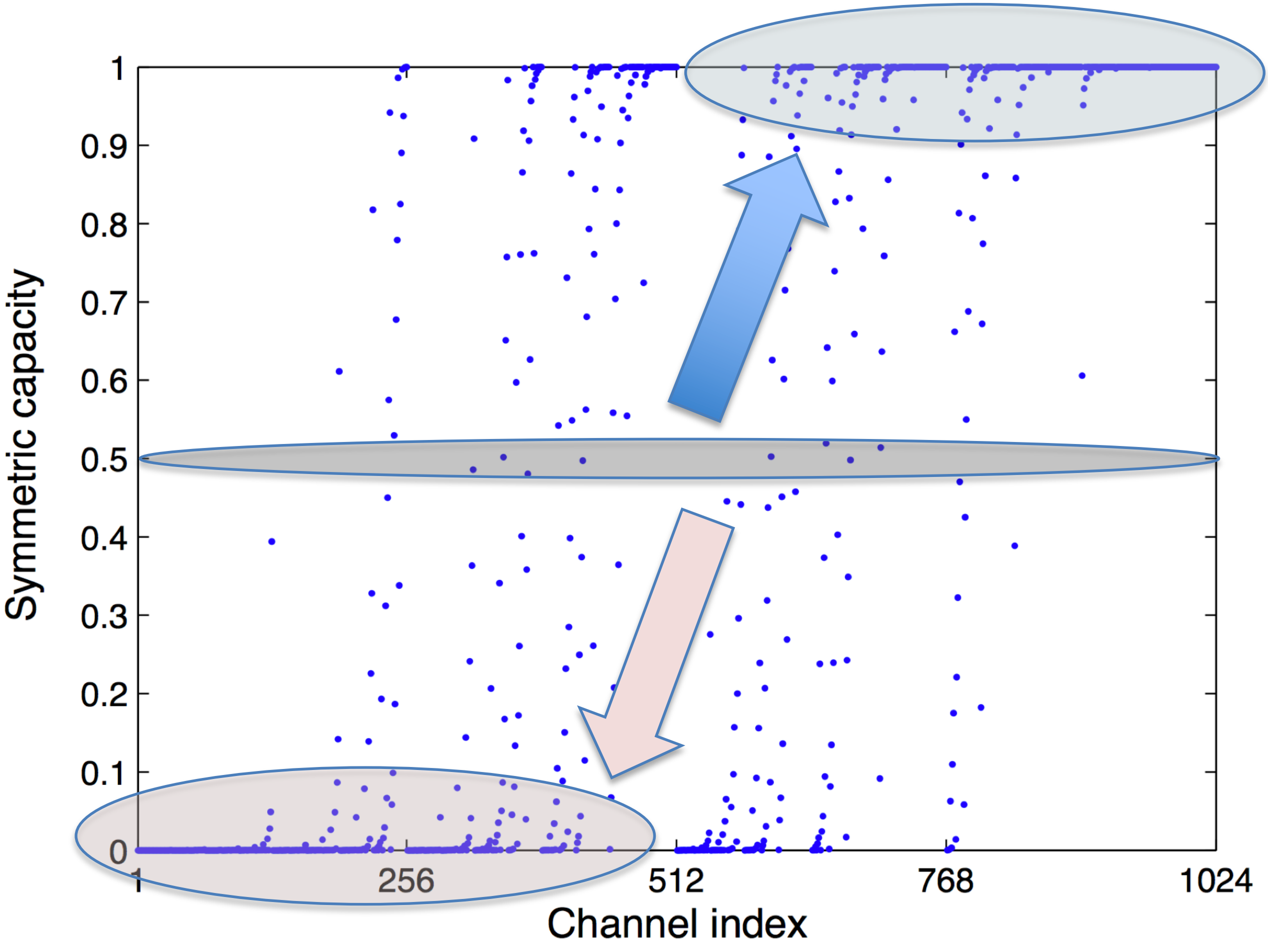


$$\boldsymbol{F} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{\otimes 4}$$

Polarization Phenomenon

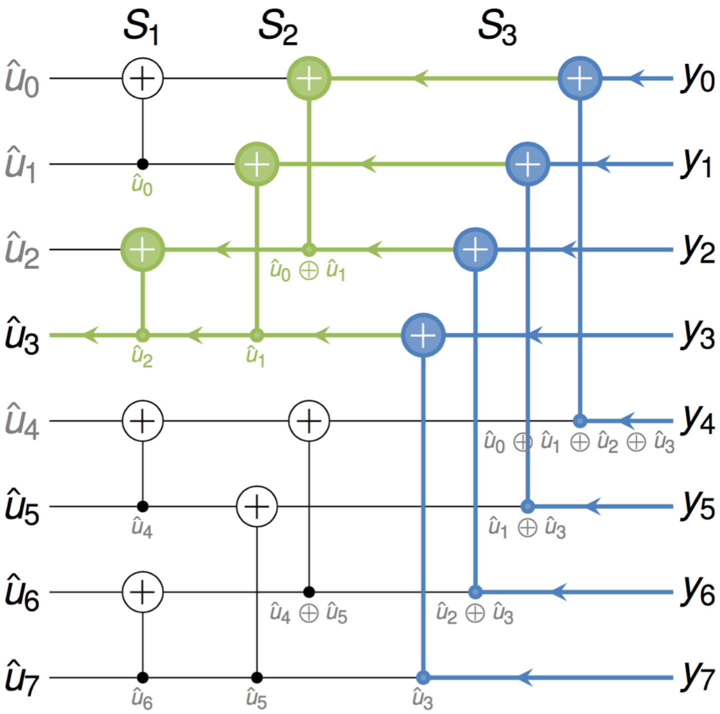
- Polar kernel polarizes messages into ‘bad’ and ‘good’ sub-channels
- Proportion of good sub-channels approaches channel capacity

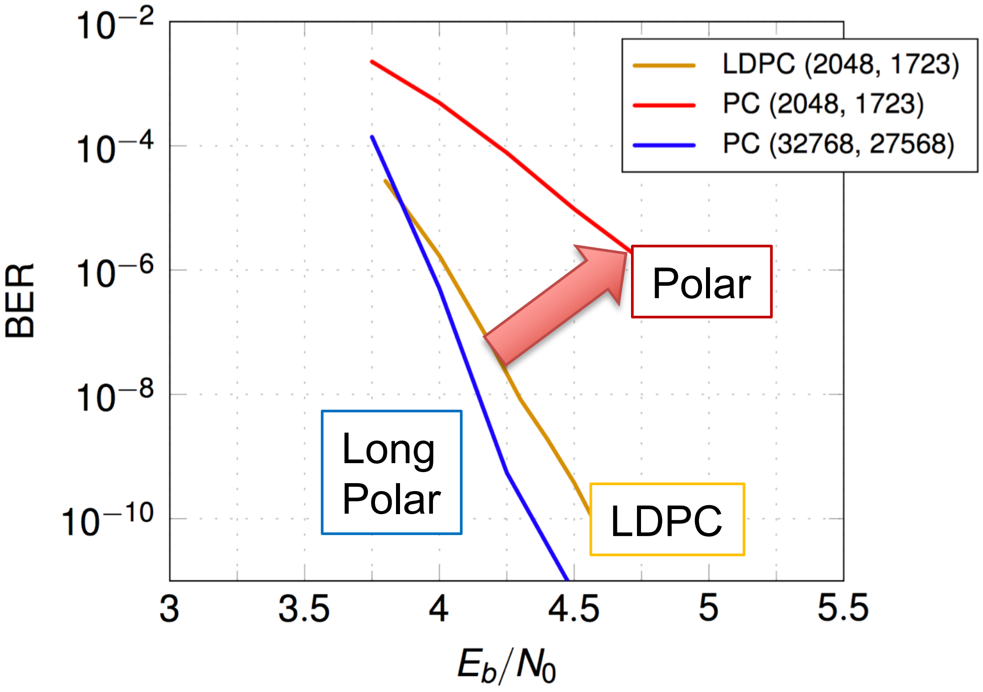




Successive Cancellation (SC) Decoding

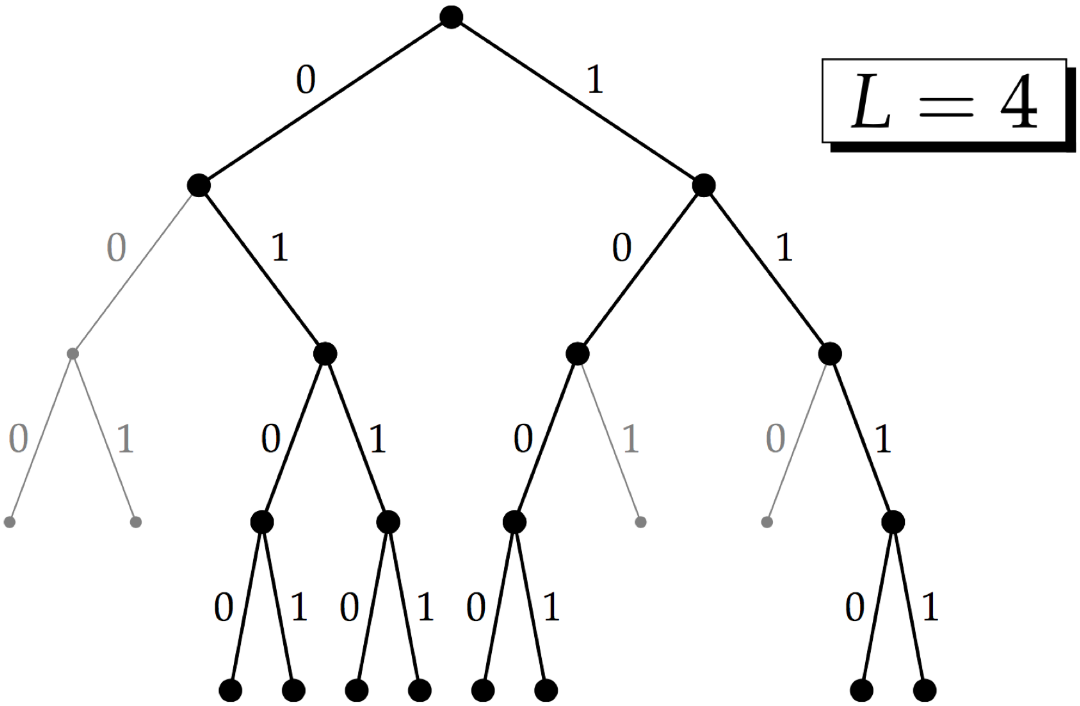
- Log-linear decoding complexity: $N \log_2(N)$
- Capacity achieving in long codes
- Disappointing performance compared to state-of-the-art LDPC codes
  - Error propagation
  - Very long codes are required: Decoding latency issue





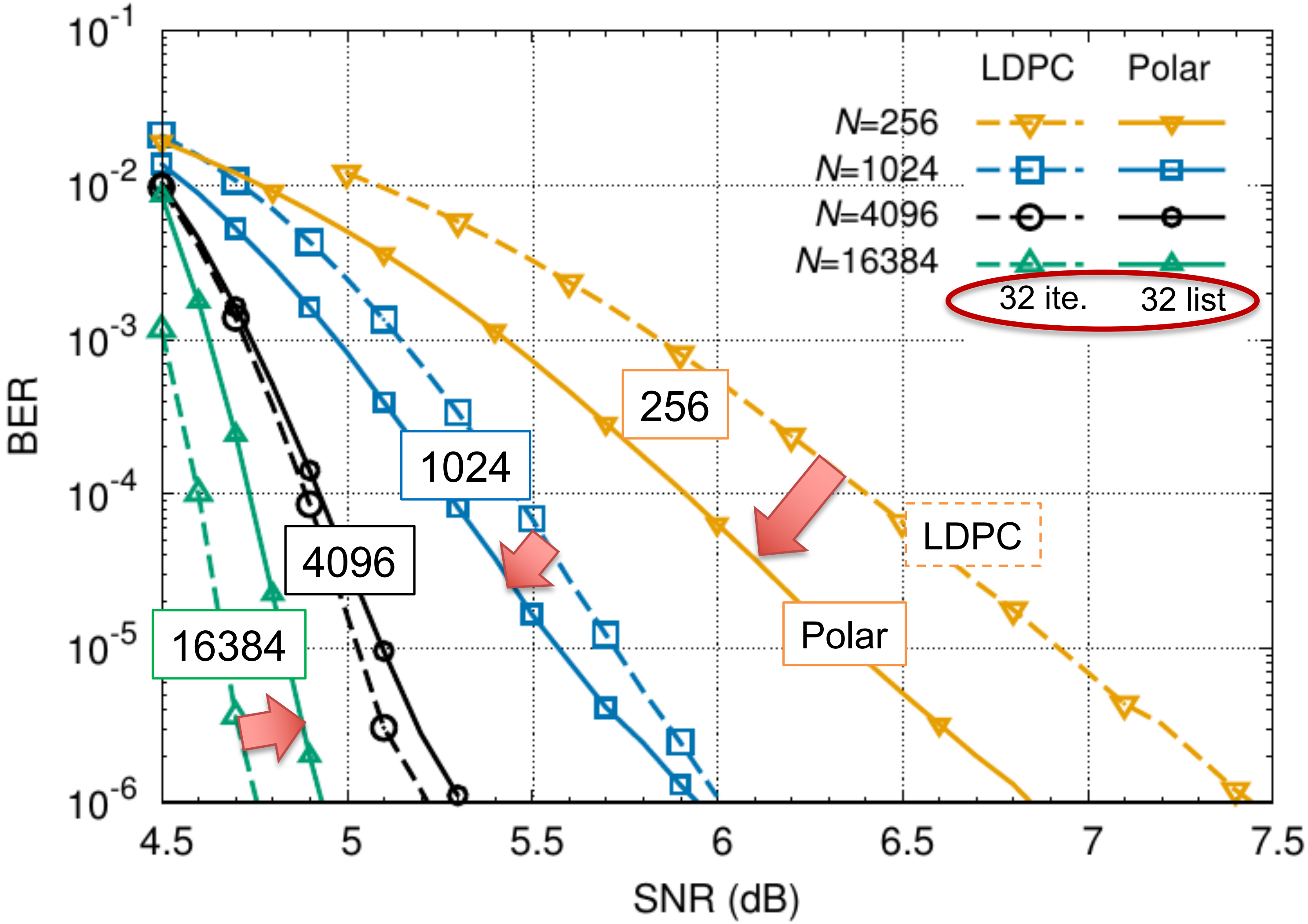
Successive Cancelation List (SCL) Decoding + CRC

- Recent breakthrough to be competitive against LDPC codes [Tal-Vardy 2015]
  - List decoding to prevent error propagation
  - Cyclic-redundancy check (**CRC**) to validate codeword in the list



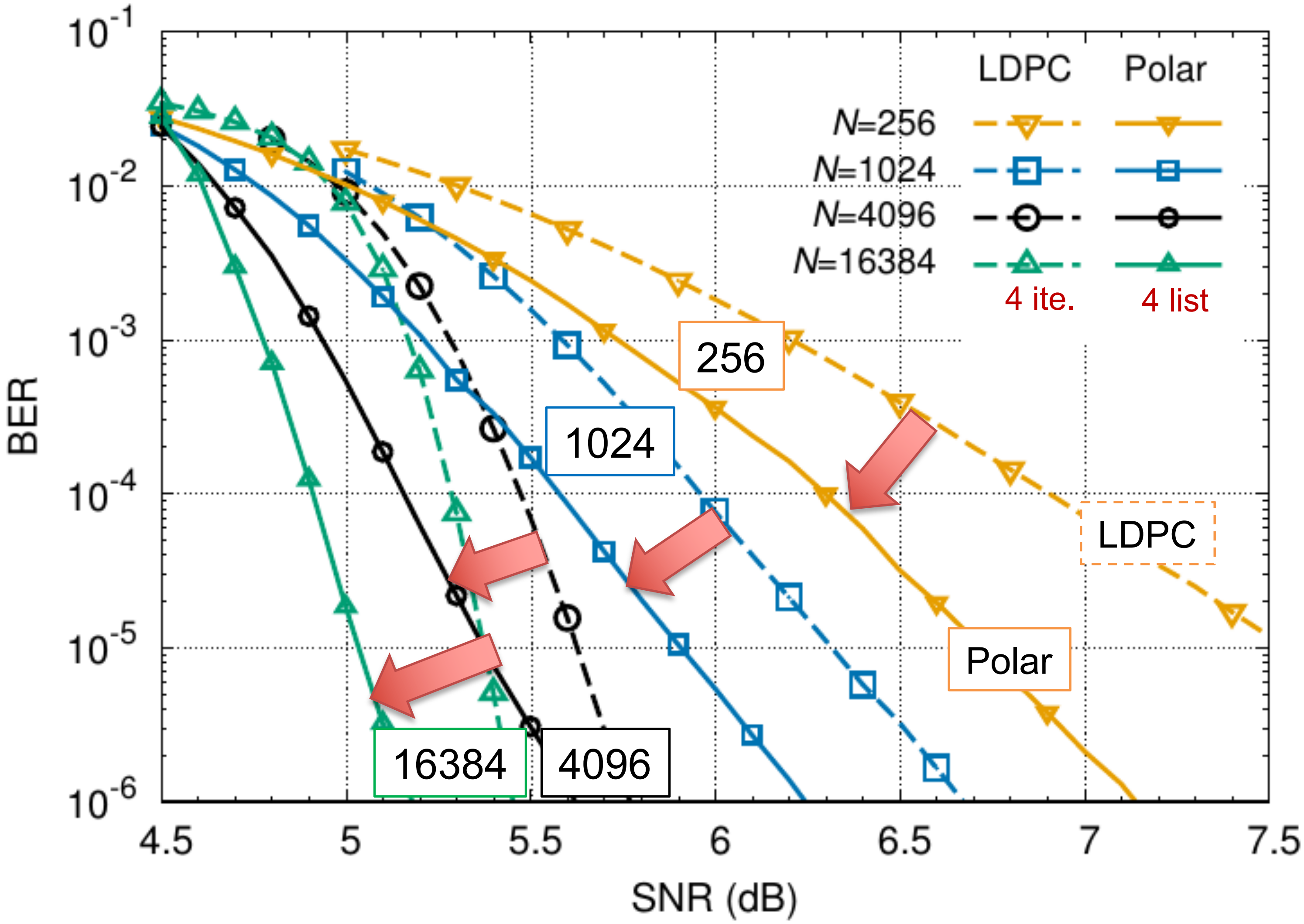
Polar vs. LDPC Codes (4QAM, List-32, Ite-32)

- Systematic polar+CRC vs. Pareto-optimal LDPC codes [KoikeAkino OFC 2017]



Polar vs. LDPC Codes (4QAM, List-4, Ite-4)

- Polar codes can outperform LDPC codes for *lower complexity* and *latency* regimes



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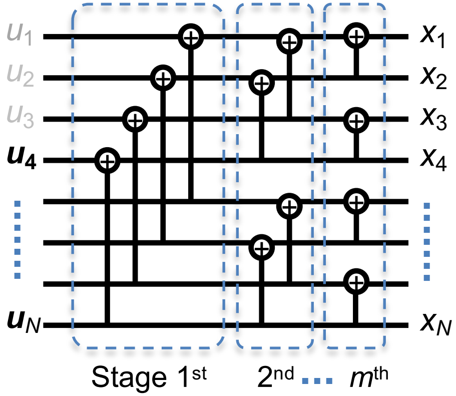
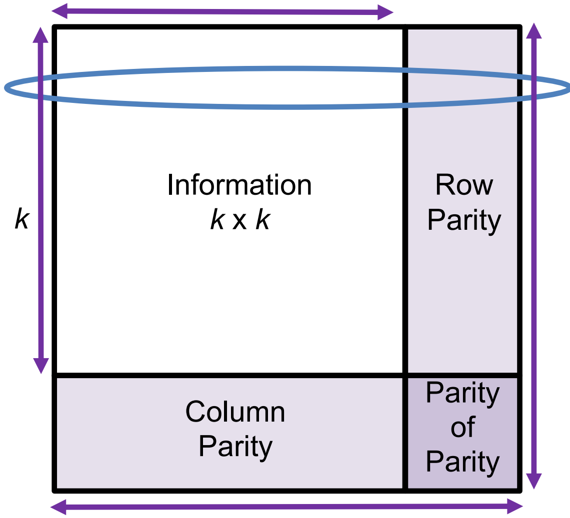
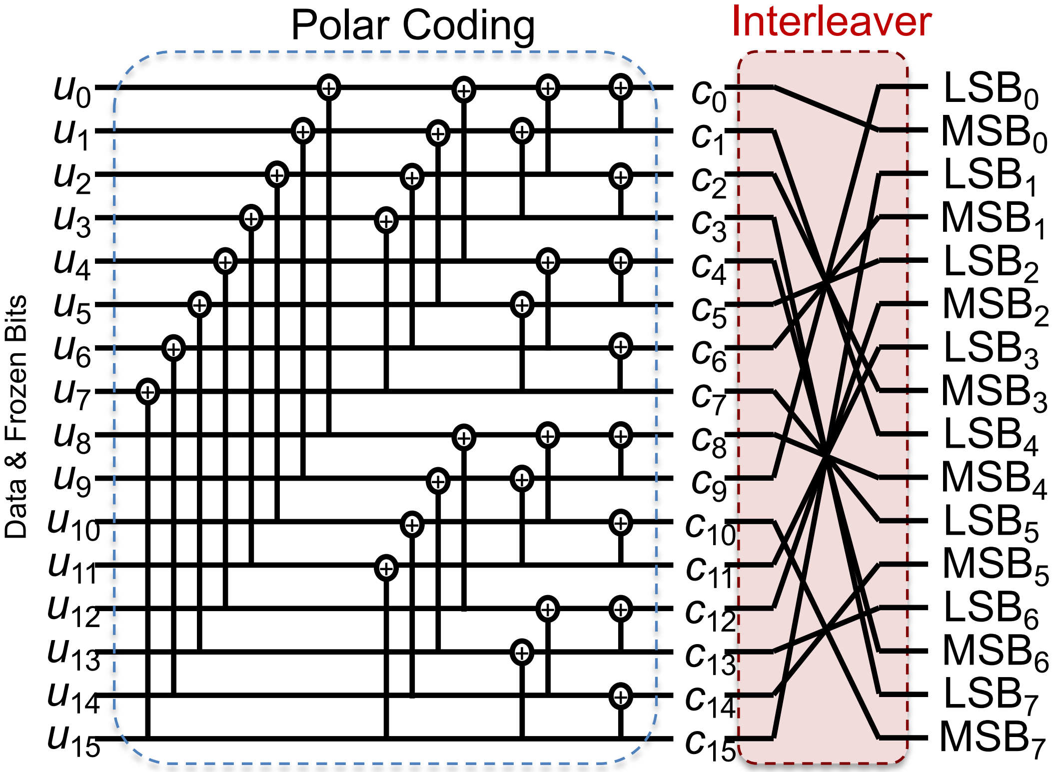
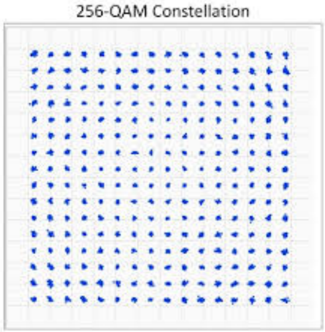


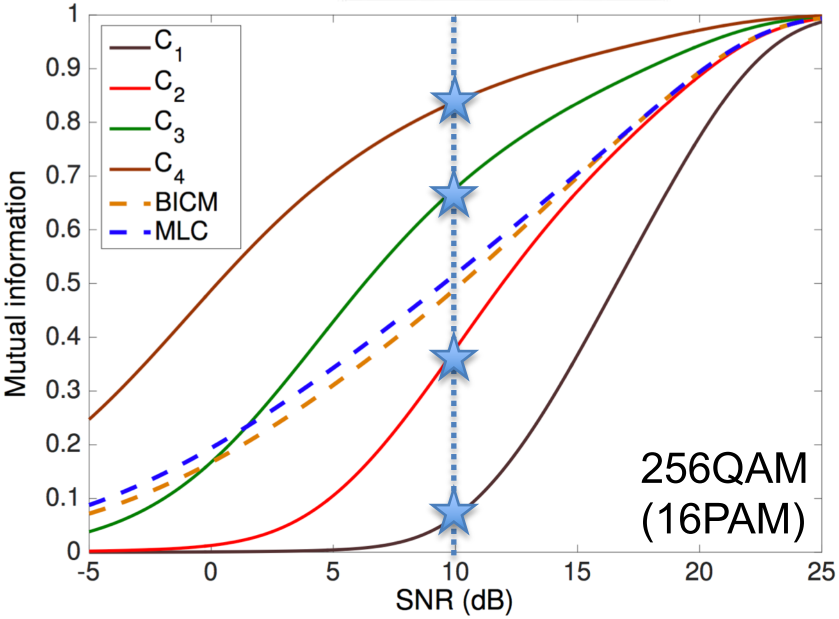


Polar-Coded BICM for High-Order QAM

- High-order modulation has non-uniform reliability for different bit-plane significance
- Interleaver is used for **BICM**
  - Interleaver does not always work for polar codes because polarization speed is affected
  - Appropriate interleaver design can improve polarization speed [KoikeAkino OFC17]







Interleaver Design

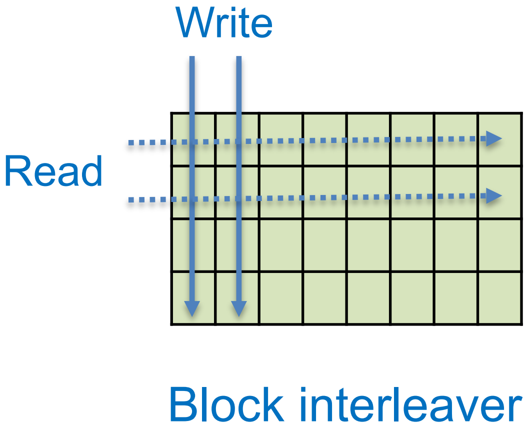
- Rectangular block interleaver
  - All combinations for the power-of-two numbers of columns and rows to design

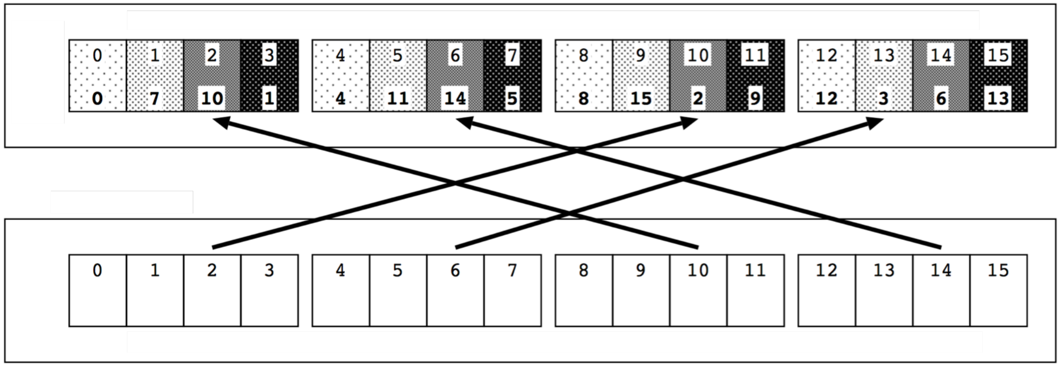
- Quadratic permutation polynomial (**QPP**) interleaver
  - Used for turbo codes in wireless communications standard
  - Maximum contention free
  - Few number of parameters to design

$$\Pi(n) = (f_0 + f_1 n + f_2 n^2) \bmod N$$

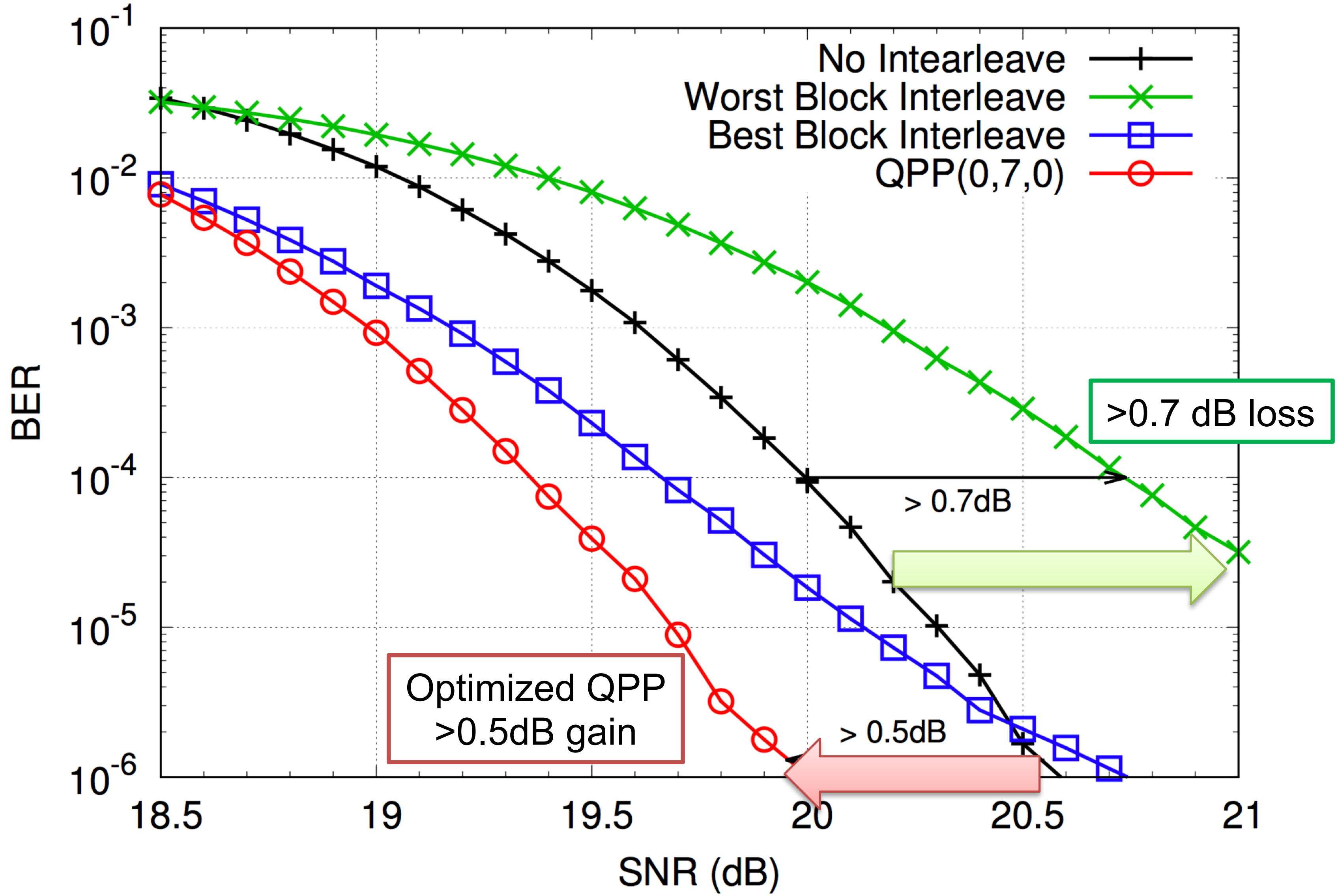
$$1 \leq f_1 \leq 71 : \text{coprime to } N$$

$$0 \leq f_2 \leq N : 0 \text{ or power of two}$$





Polar-Coded 256QAM

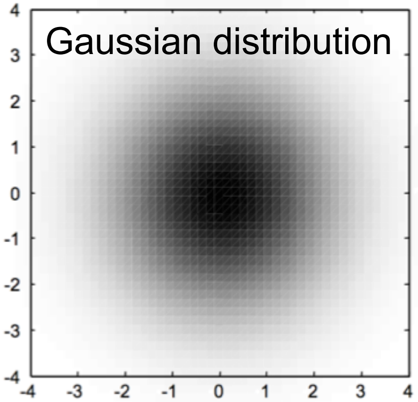


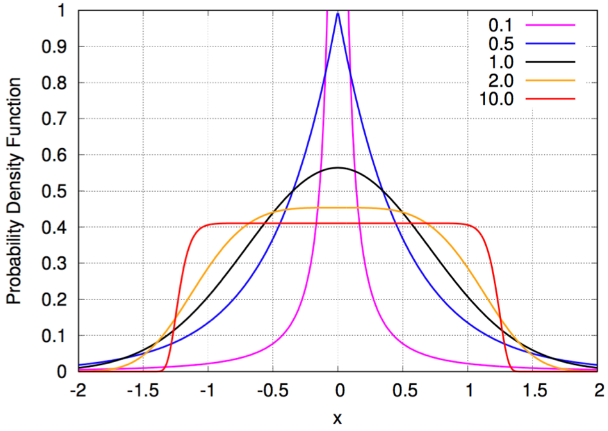
>0.7 dB loss

Optimized QPP
>0.5dB gain

Non-Uniform Multi-Level QAM

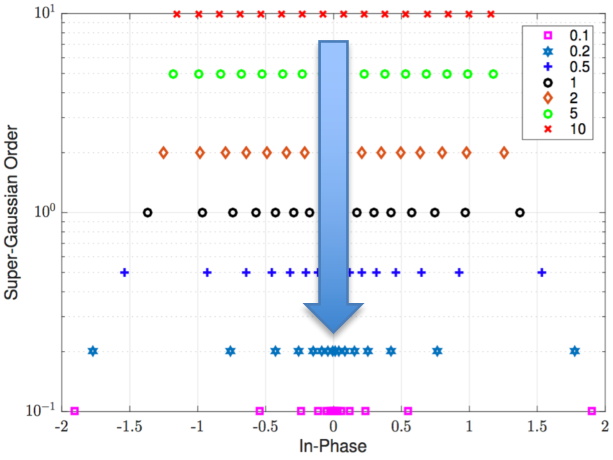
- Uniform QAM has shaping loss from optimal Gaussian signal distribution
- We propose geometric shaping for QAM with super-Gaussian non-uniformity

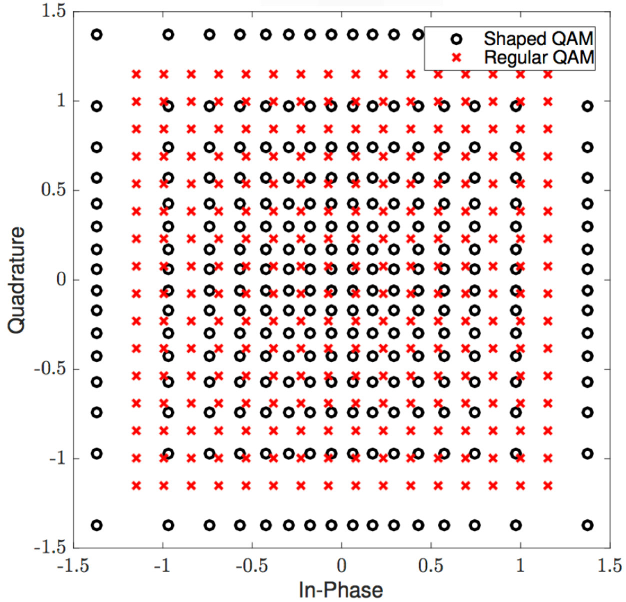


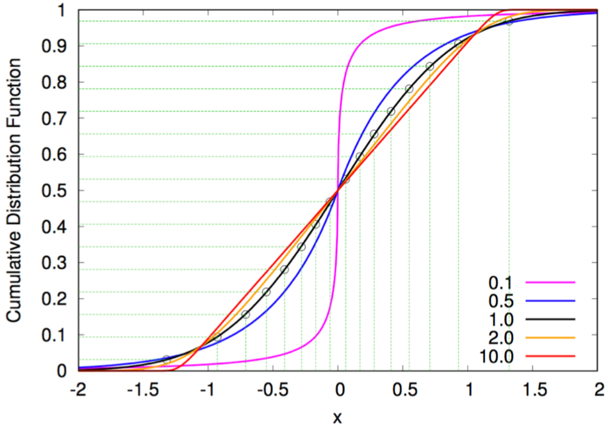


(a) PDF

$$f(x) = \frac{1}{2\Gamma\left(1+\frac{1}{2\gamma}\right)\sqrt{2\sigma^2}} \exp\left(-\left(\frac{x^2}{2\sigma^2}\right)^{\gamma}\right)$$

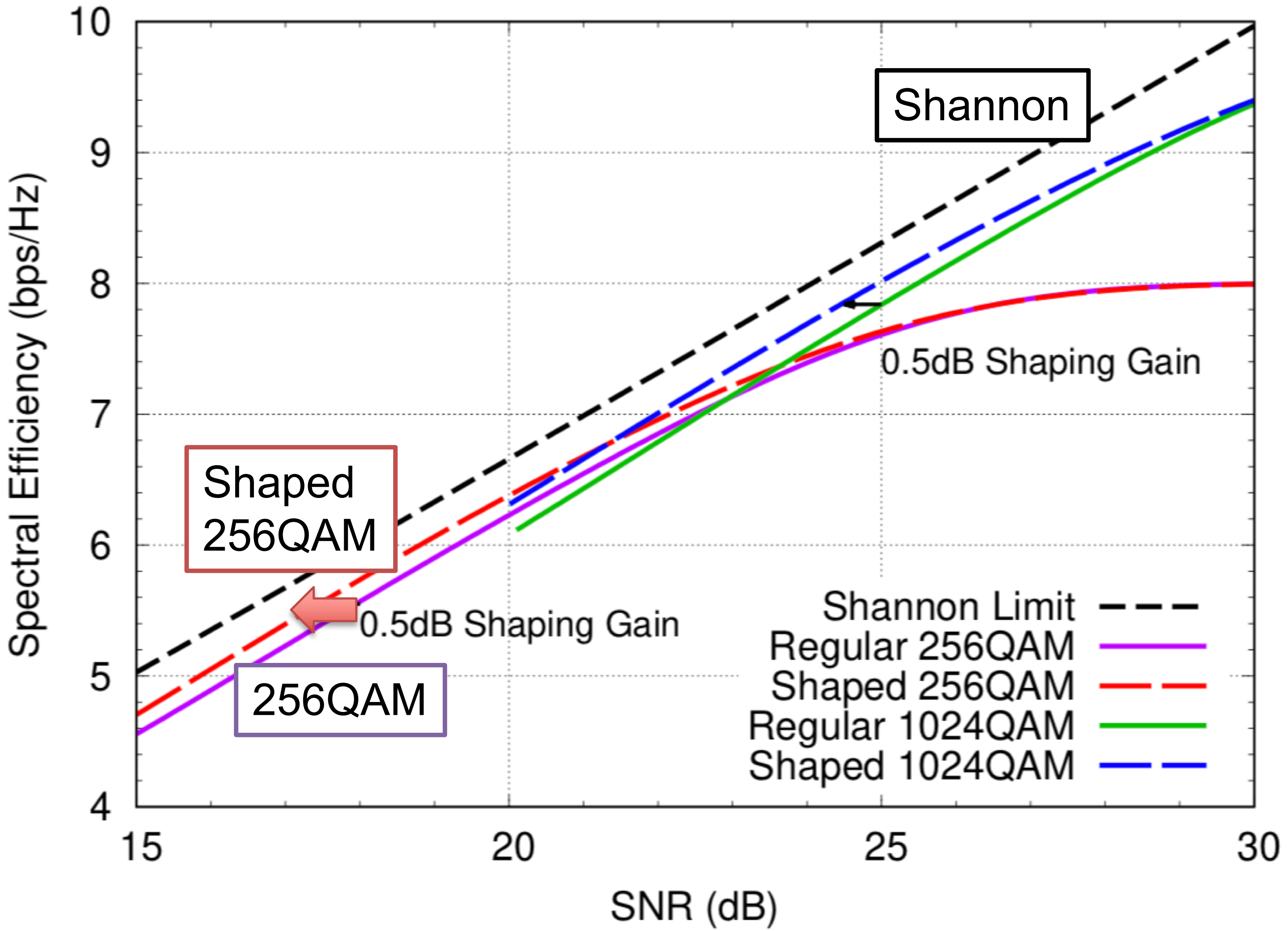




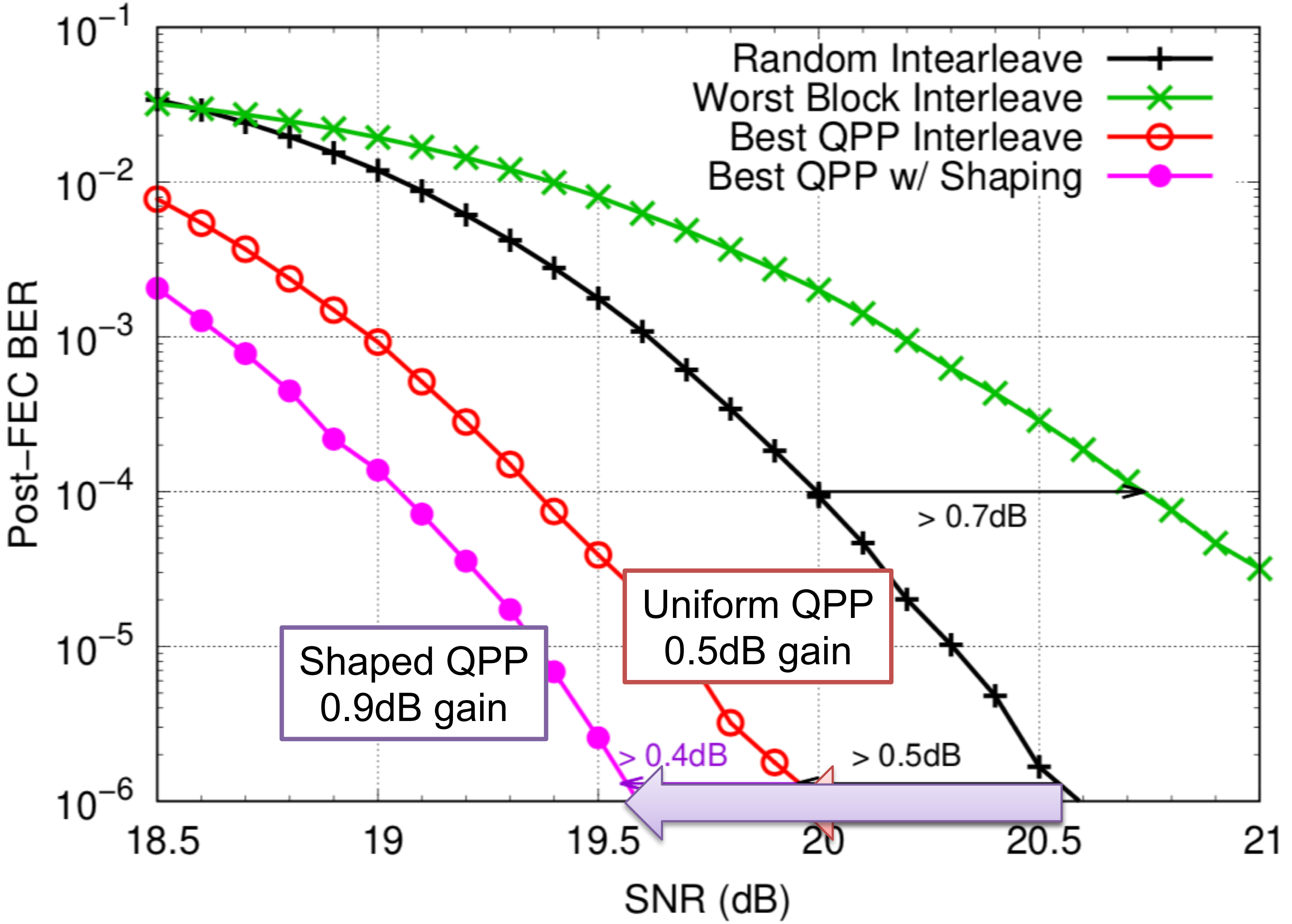


(b) CDF

Shaping Gain of Non-Uniform QAM



Polar-Coded 256QAM with Constellation Shaping

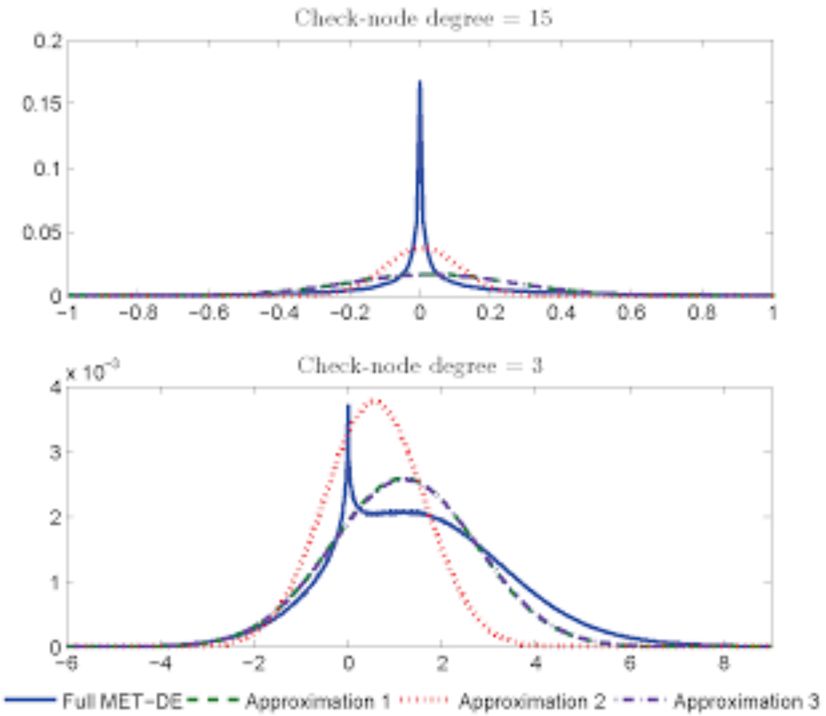


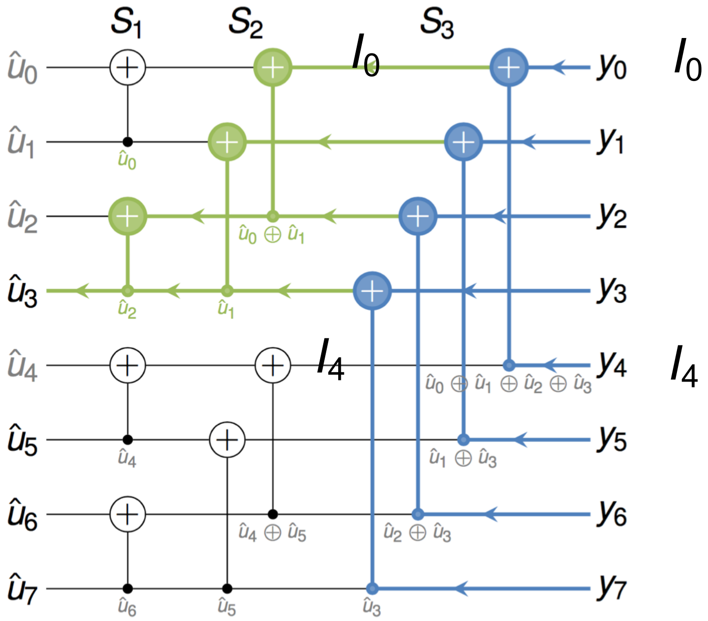
Shaped QPP
0.9dB gain

Uniform QPP
0.5dB gain

Design via EXIT (Extrinsic Information Transfer)

- Frozen bit location can be designed by density evolution (**DE**), while DE is cumbersome for non-uniform bit reliability
- Gaussian approximation (**GA**) can simplify DE, assuming input and output messages as Gaussian
- **EXIT** chart does not impose Gaussian assumption, thus more accurate than GA



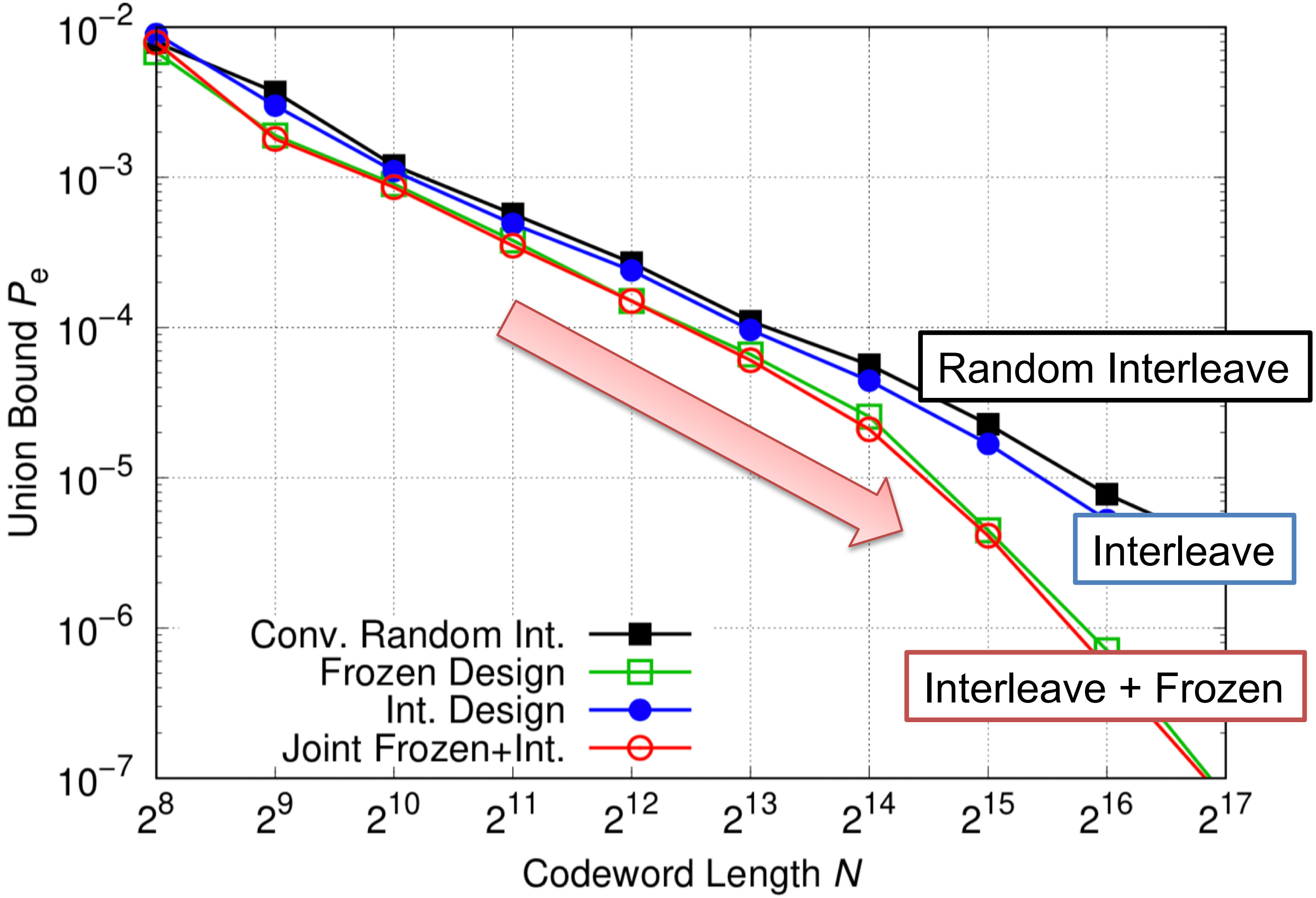


$$\mathcal{I}_{r_{\mathrm{U}}}^{[l-1]} = 1 - J\left(\sqrt{\left[J^{-1}(1-\mathcal{I}_{r_{\mathrm{U}}}^{[l]})\right]^2 + \left[J^{-1}(1-\mathcal{I}_{r_{\mathrm{L}}}^{[l]})\right]^2}\right),$$

$$\mathcal{I}_{r_{\mathrm{L}}}^{[l-1]} = J\left(\sqrt{\left[J^{-1}(\mathcal{I}_{r_{\mathrm{U}}}^{[l]})\right]^2 + \left[J^{-1}(\mathcal{I}_{r_{\mathrm{L}}}^{[l]})\right]^2}\right),$$

Polarization Rate Boosting (256QAM)

- Joint interleaver and frozen bit location design can boost polarization speed



Random Interleave

Interleave

Interleave + Frozen

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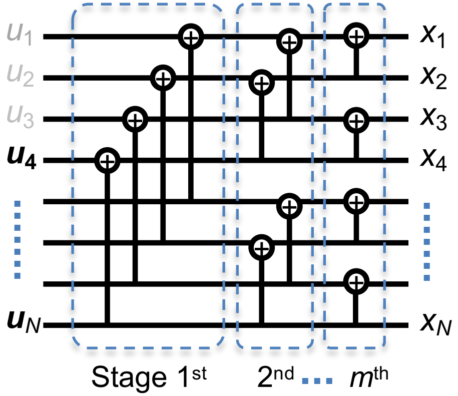
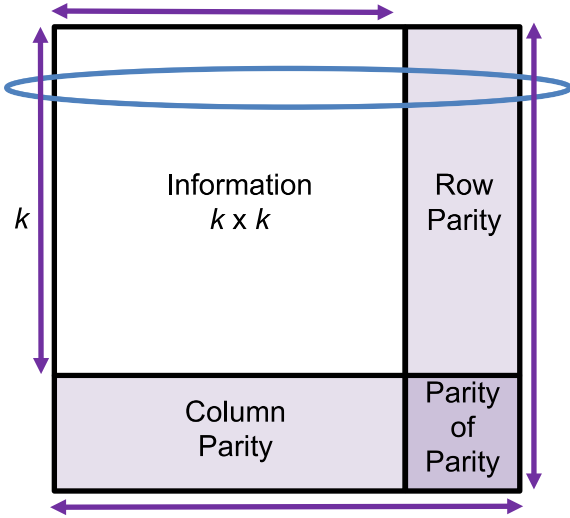
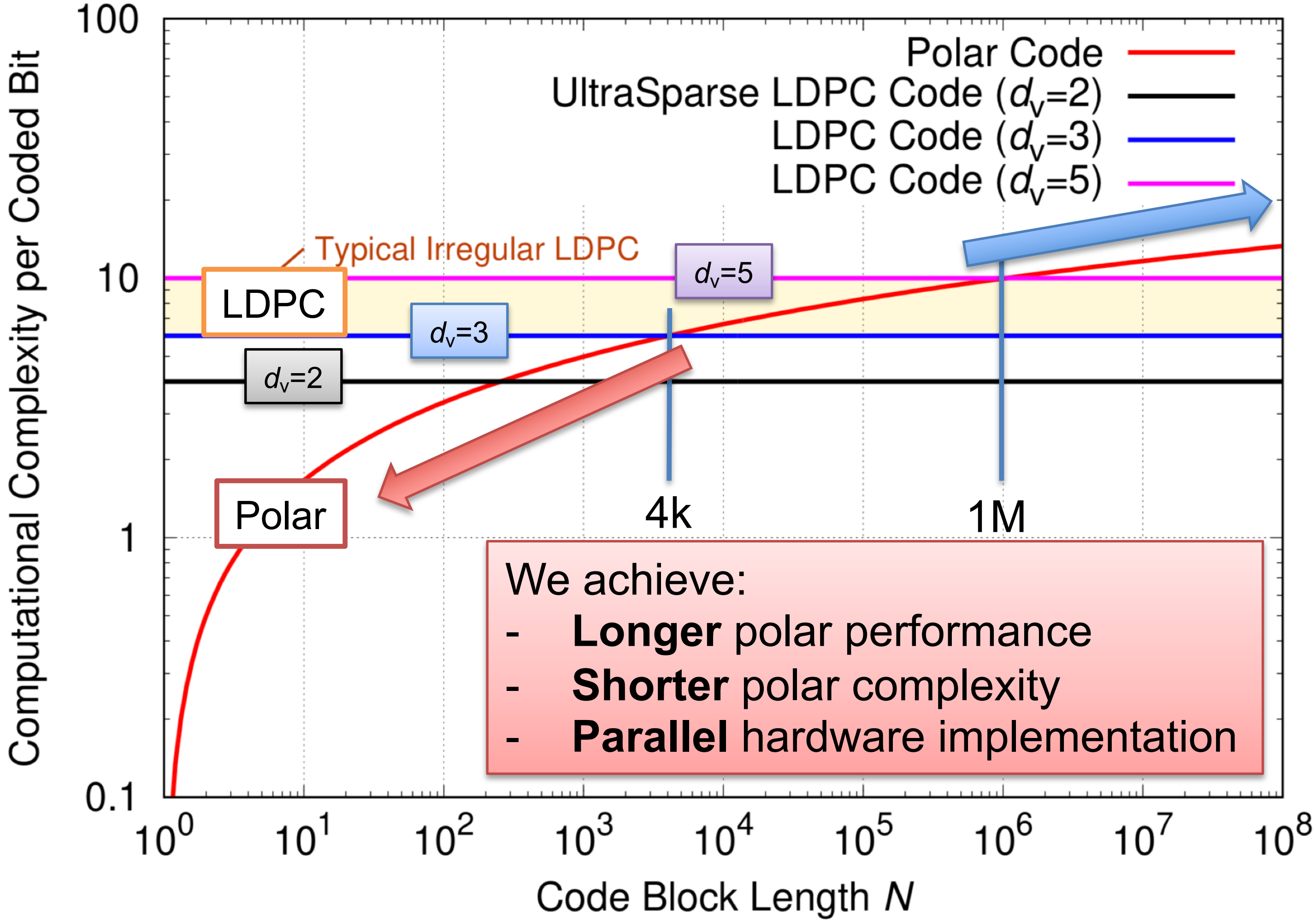




Computational Complexity

- Polar decoding requires nonlinear complexity: $L\,N\log_2(N)/2$
- LDPC BP decoding has linear complexity: $2\,I\,d_v\,N$

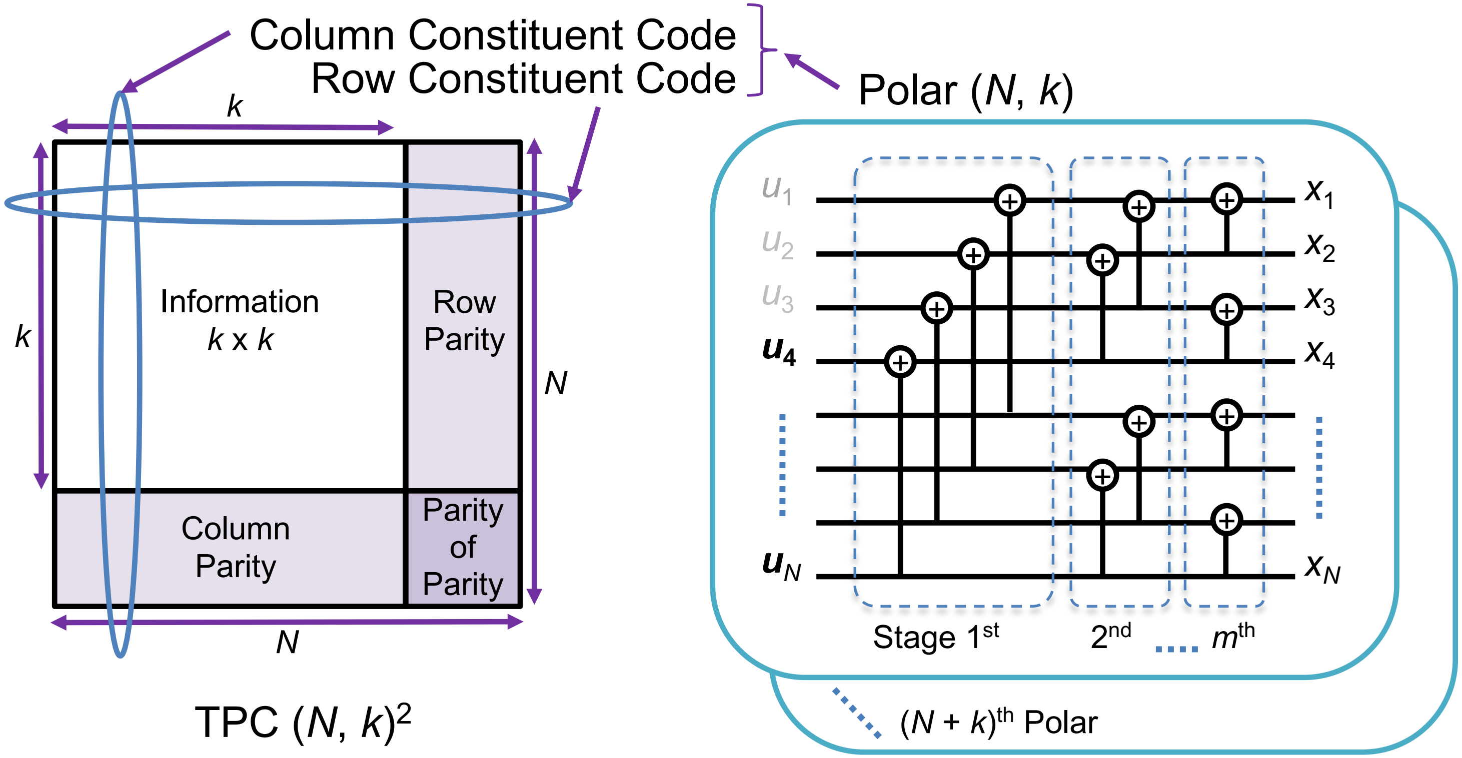


We achieve:

- **Longer** polar performance
- **Shorter** polar complexity
- **Parallel** hardware implementation

Turbo Product Codes (TPC)

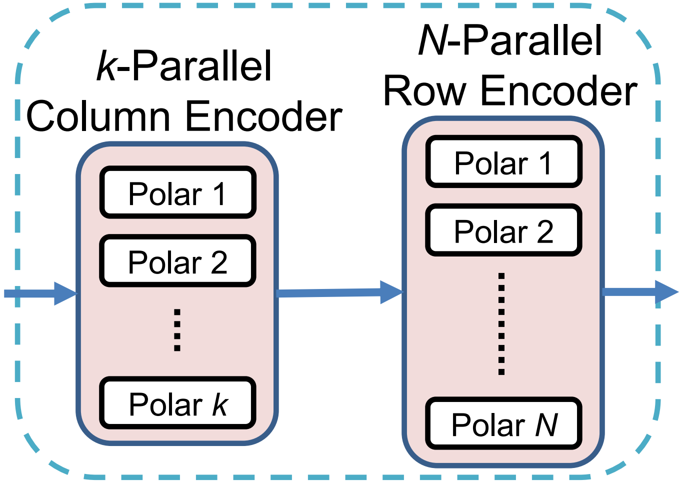
- Resolve the issue of parallelism in SCL decoding by polar product codes [OFC18]
- Highly parallel encoding and decoding are enabled; **256-times faster throughput**
- Polar-TPC outperforms conventional BCH-TPC by **0.5dB**

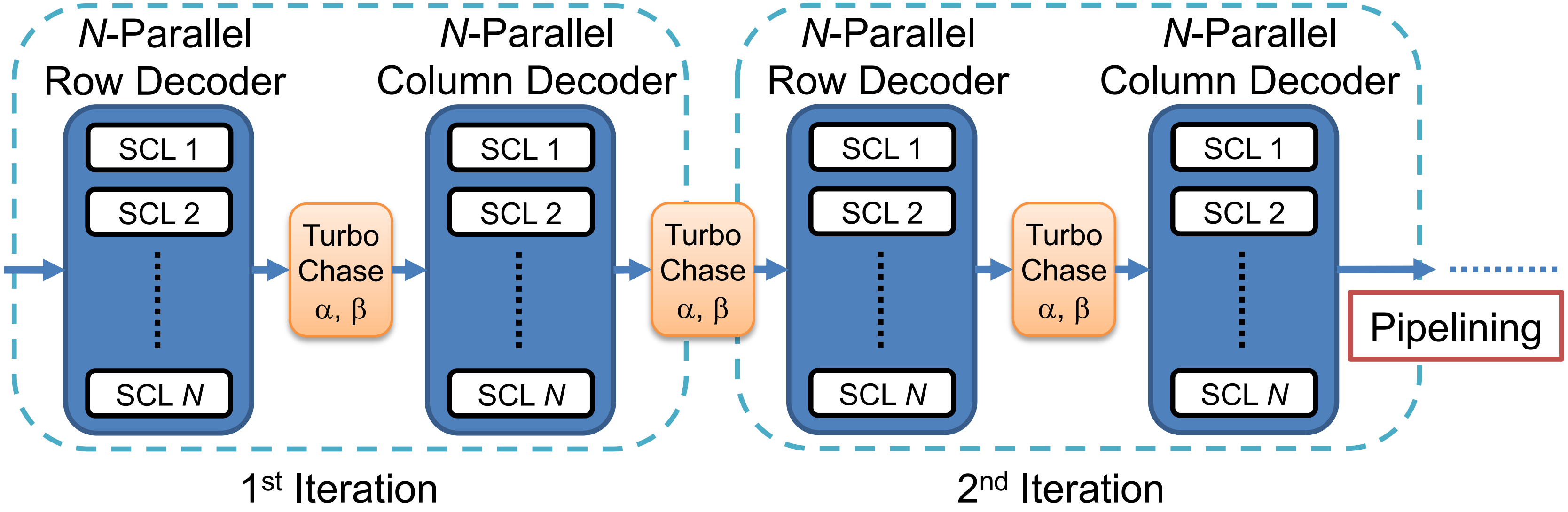


TPC $(N, k)^2$

Polar-TPC Encoding and Decoding

- Massively parallel and pipeline processing
- Soft-decision output is generated by Chase algorithm [Pyndiah TCOM1998], exploiting survival lists in SCL decoder
- *N*-times faster decoding throughput is possible
  - For $(256, 239)^2$, we achieve **256-times speed-up**





Related Works: Concatenated Polar Codes

- With **Hamming codes**
  - M. Seidl and J. B. Huber, “Improving successive cancellation decoding of polar codes by usage of inner block codes,” in *Proc. Int. Symp. Turbo Codes Iterative Inf. Process.*, pp. 103–106, Brest, France, Sep. 2010.
- With **LDPC codes**
  - J. Guo, M. Qin, A. Guillen i Fabregas, and P. H. Siegel, “Enhanced belief propagation decoding of polar codes through concatenation,” in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, pp. 2987–2991, Honolulu, HI, June 2014,
  - A. Eslami and H. Pishro-Nik, “On finite-length performance of po- lar codes: Stopping sets, error floor, and concatenated design,” *IEEE Trans. Commun.*, vol. 61, no. 3, pp. 919–929, Mar. 2013.
  - Y. X. Zhang and A. Liu, “Polar-LDPC concatenated coding for the AWGN wiretap channel,” *IEEE Commun. Lett.*, vol. 18, no. 10, pp. 1683–1686, Oct. 2014.
- With **BCH codes**
  - Y. Wang, K. R. Narayanan, and Y.-C. Huang, “Interleaved concate- nations of polar codes with BCH and convolutional codes,” *IEEE J. Sel. Areas Commun.*, vol. 34, no. 2, pp. 267–277, Feb. 2016.
- With **convolutional codes**
  - Q. Zhang, A. Liu, Y. Zhang, and X. Liang, “Practical design and decoding of parallel concatenated structure for systematic polar codes,” *IEEE Trans. Commun.*, vol. 64, no. 2, pp. 456–466, Feb. 2016.
- With **Reed-Solomon codes**
  - H. Mahdavifar, M. El-Khamy, J. Lee, and I. Kang, “Performance limits and practical decoding of interleaved Reed-Solomon polar concatenated codes,” *IEEE Trans. Commun.*, vol. 62, no. 5, pp. 1406–1417, May 2014.

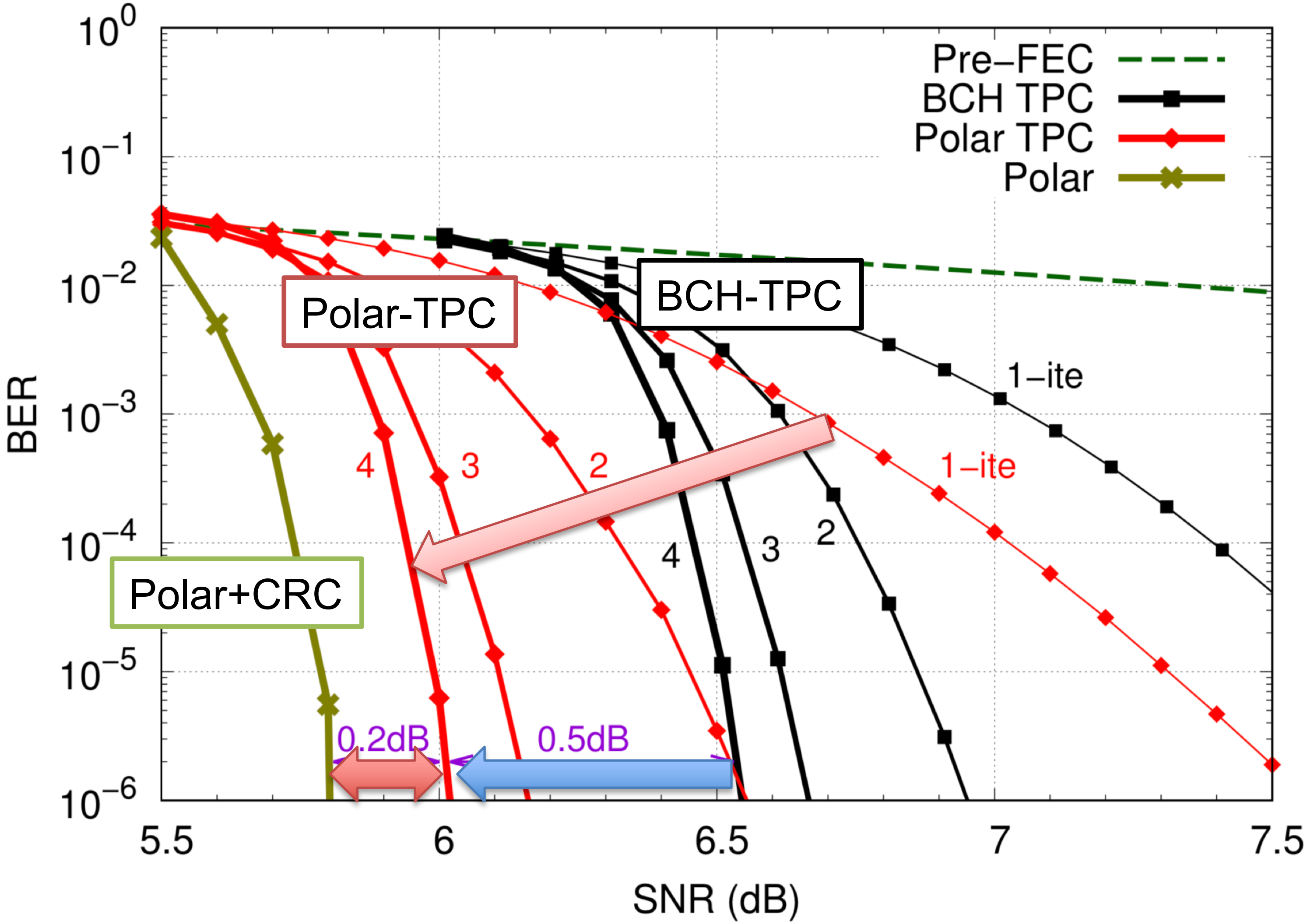
No Polar-TPC

BP decoding based

We propose SCL-based TPC

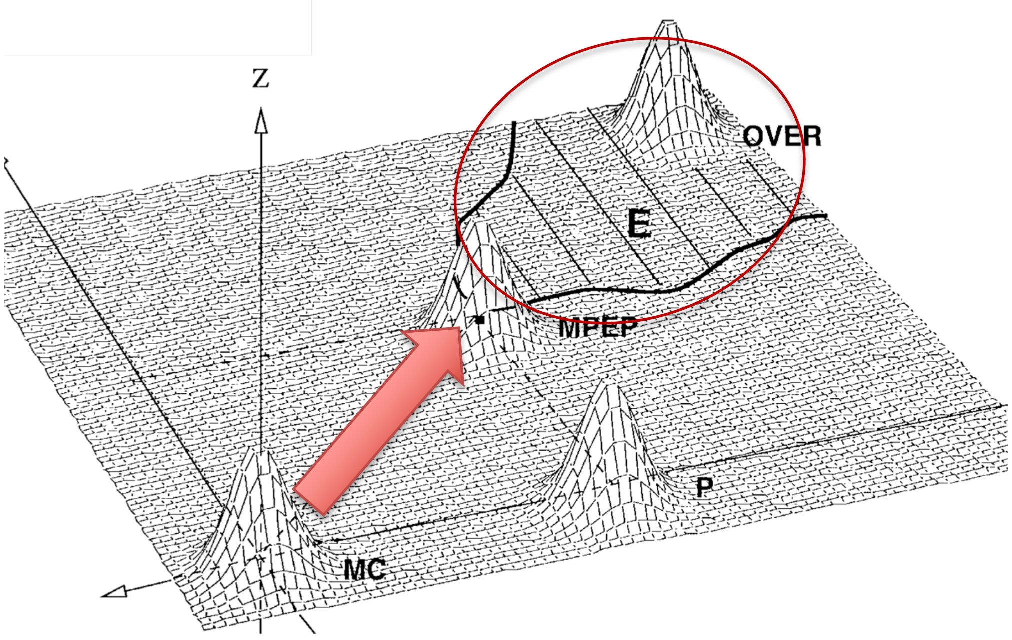
Polar-TPC Performance

- Polar-TPC$(256, 239)^2$ vs BCH-TPC$(256, 239)^2$ vs Polar$(256^2, 239^2+16)$+CRC16



Error Floor Analysis with Importance Sampling (IS)

- Importance sampling (IS) can reduce the required number of simulation runs to achieve high confidence compared to Monte-Carlo (MC) via weighted sample mean
- IS has been used for BCH-TPC analysis:
  - M. Ferrari, S. Bellini, Importance sampling simulation of turbo product codes, ICC, 2001



$$\hat{P}_{\mathrm{IS}} = \frac{1}{n}\sum_{i=1}^{n} 1_E(\boldsymbol{x}_i)\, w(\boldsymbol{x}_i)$$

$$w(Y^{(i)}|\mu,\sigma^2) = \frac{\exp(-\frac{1}{2\sigma^2}[\sum_{k=1}^{a}(Y_k^{(i)}-1)^2])}{\exp(-\frac{1}{2\sigma^2}[\sum_{k=1}^{a}(Y_k^{(i)}-(1-\mu))^2])},$$

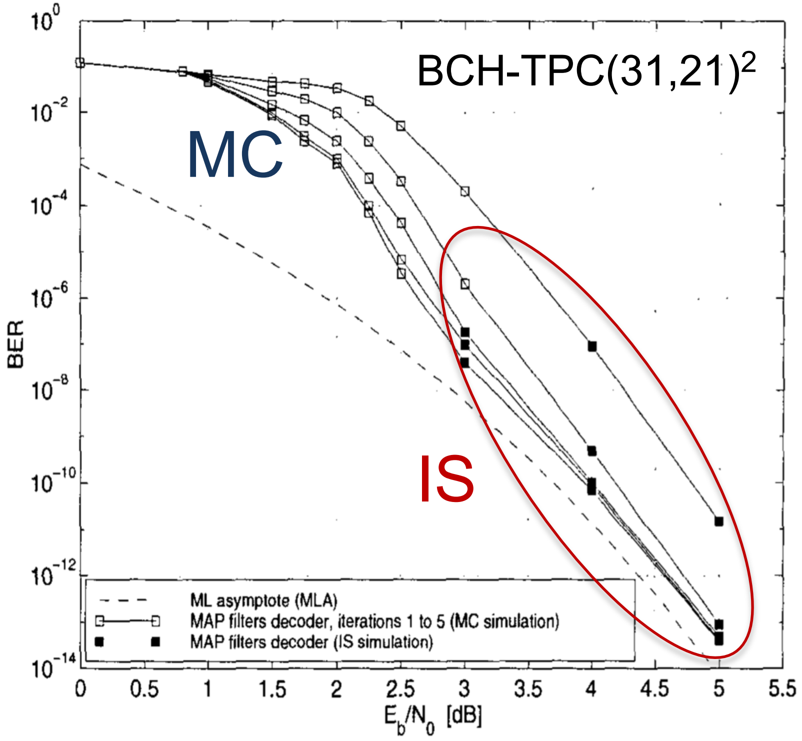
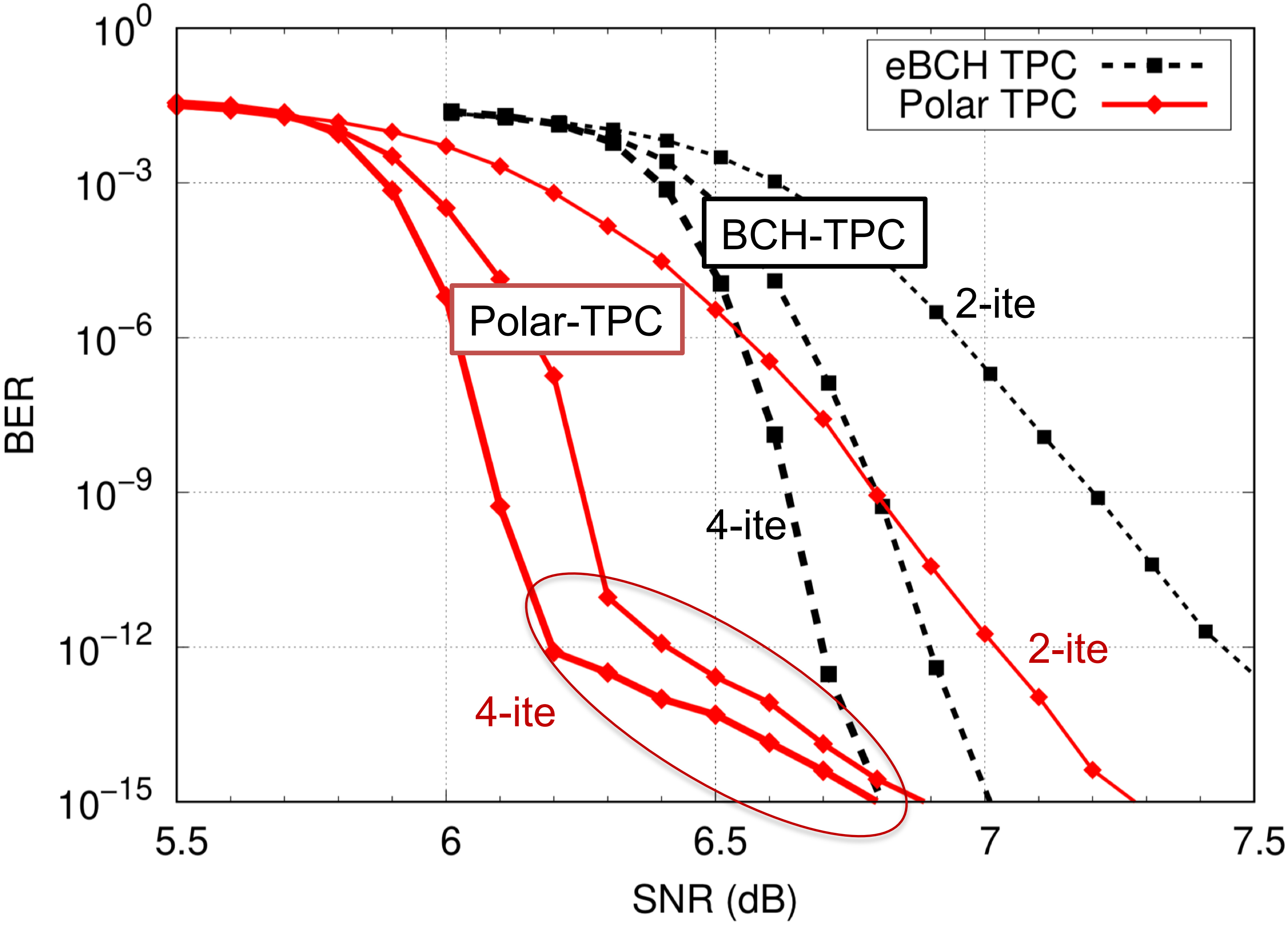


Fig. 3. Product code $BCH(31,21,5) \times BCH(31,21,5)$: MLA, and "partial factor" MAP-filters decoder

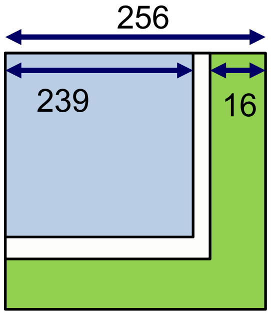
BER Performance via IS

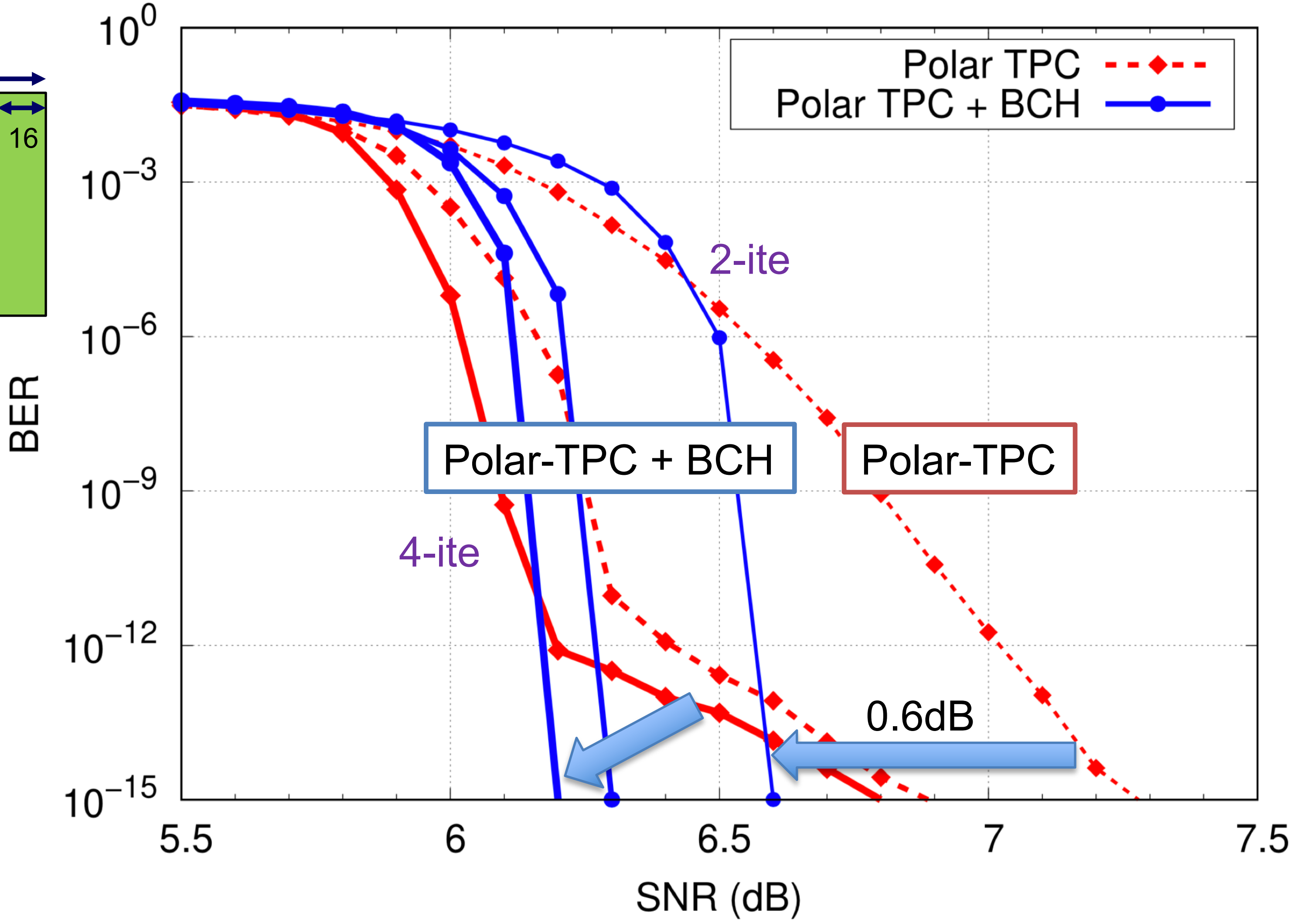
- Error floor was observed for polar-TPC(256, 239)^2



BER Performance via IS: Error Floor Mitigation

- Error floor was removed with polar-TPC$(256, 240)^2$ + BCH$(240^2, 239^2)$





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- EXIT chart design method
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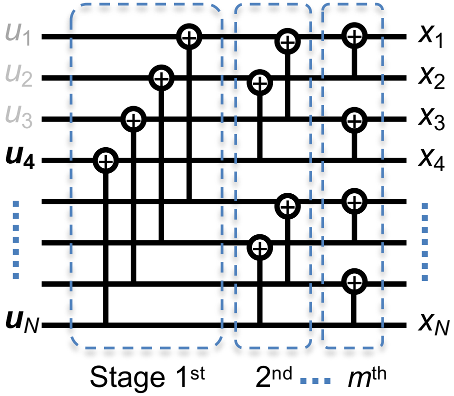
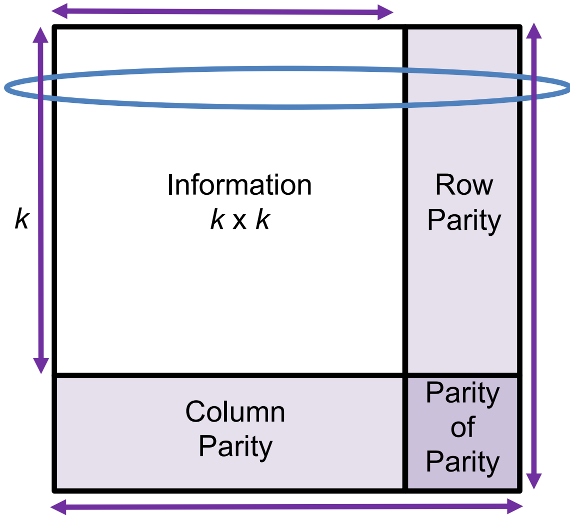




Information
k x k

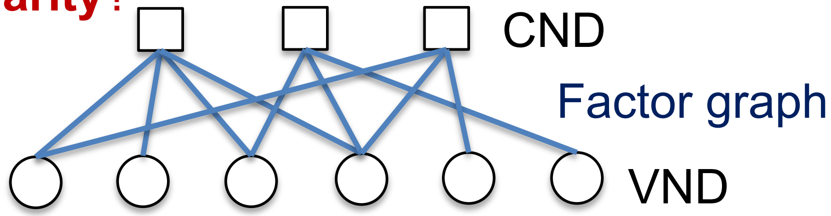
Row Parity

Column Parity

Parity of Parity

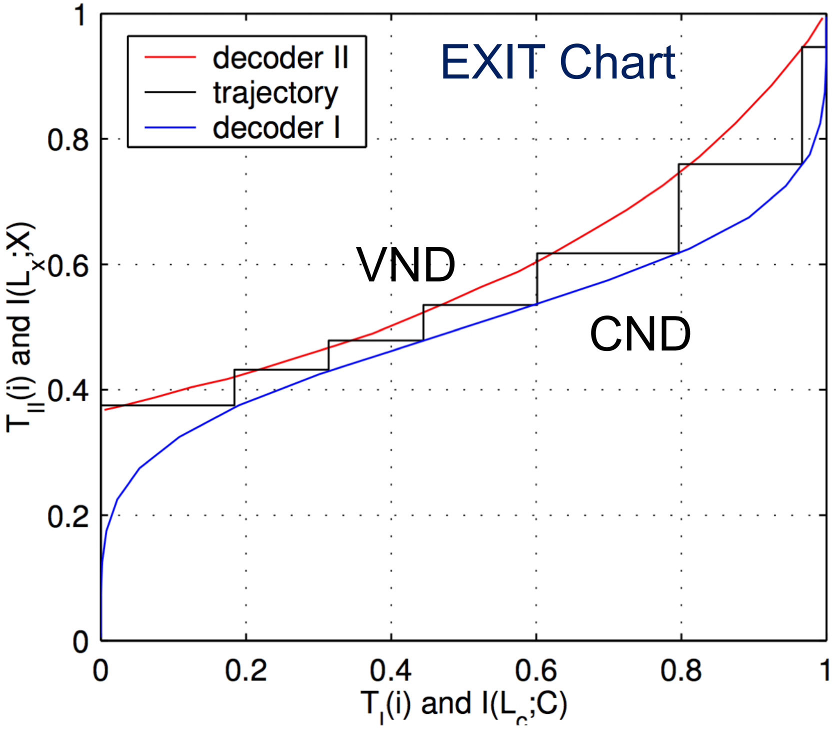
Irregular Polar Coding

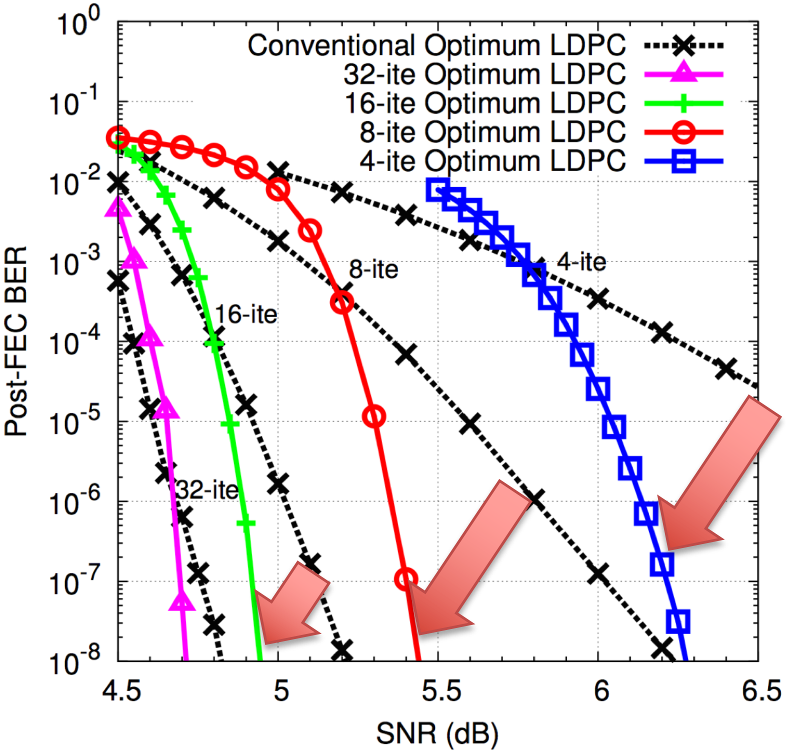
- For LDPC codes, it is well-known that ***irregular*** degree distribution can significantly improve performance over regular counterparts
- What happens for **polar coding** with **irregularity**?



Parity check

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

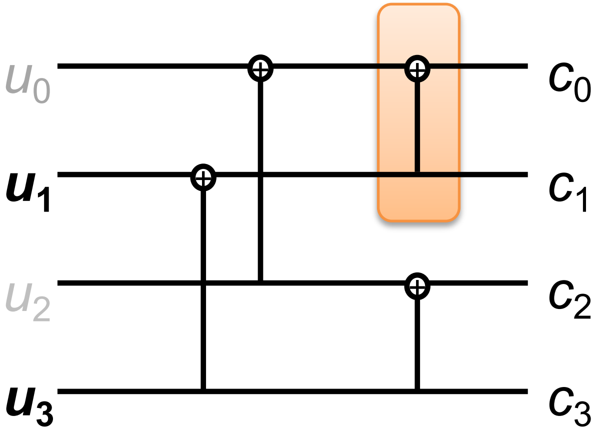




Irregular Pruning of Polarization Units

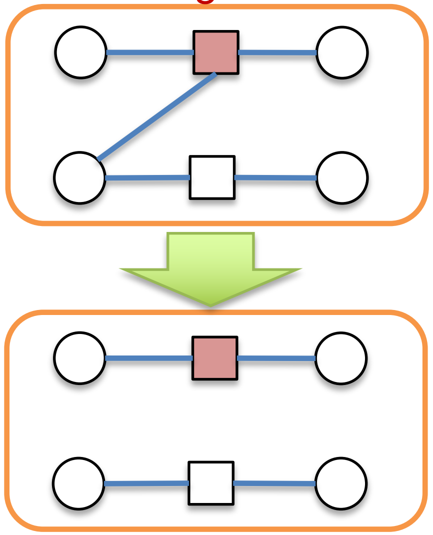
- We proposed to *inactivate* polarization units in an irregular fashion [KoikeAkino ECOC2017, GLOBECOM2017]
- We can *reduce*
  - the computational **complexity** for both encoding and decoding; 30%~80%
  - the decoding **latency** of SCL; 25%~95%
  - the bit error rate (**BER**); a marginal gain

Polarization Unit



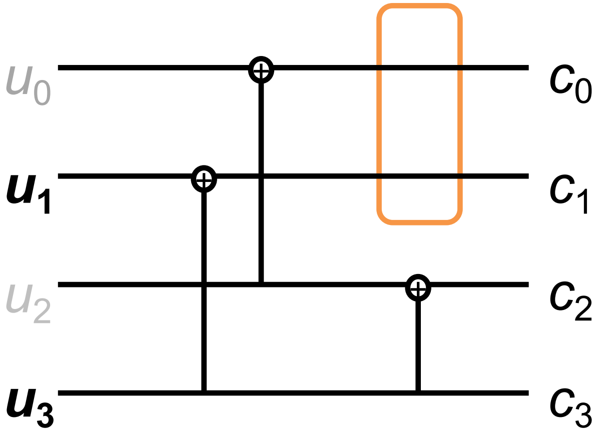
Factor Graph

Degree-3



Degree-2

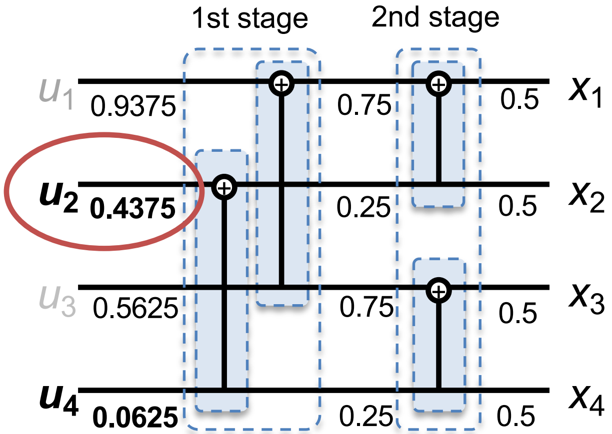
No computation



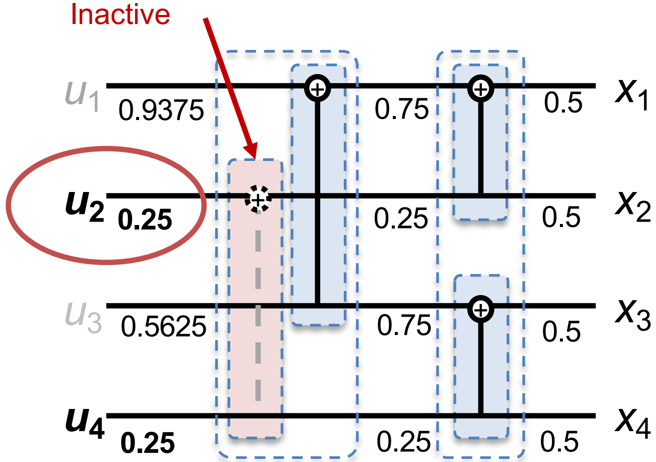
Why Irregular Polarization Helps?

- Impact of inactivated polarization units

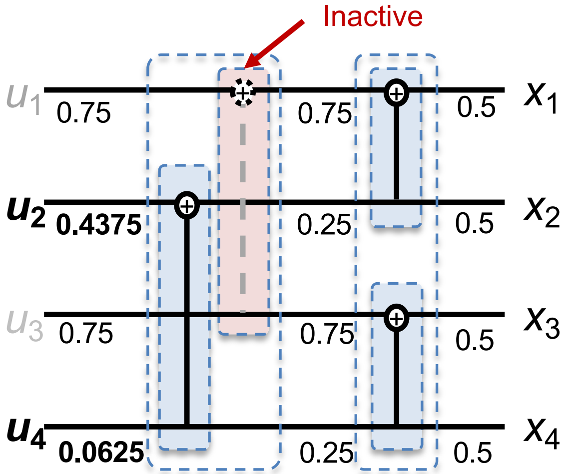
Regular Polar Code: UB = 0.2432



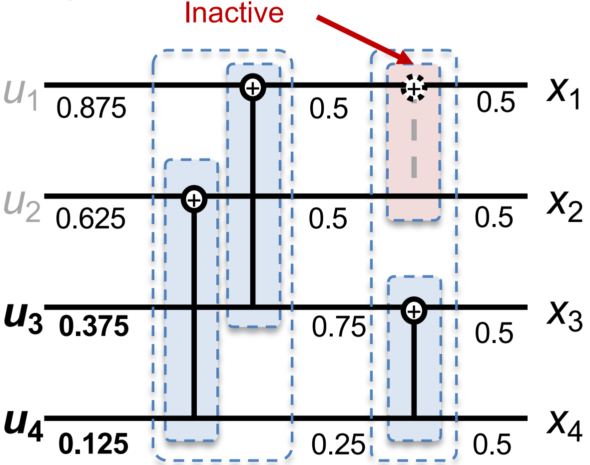
Irregular Polar Code: UB = **0.2344**



Irregular Polar Code: UB = 0.2432

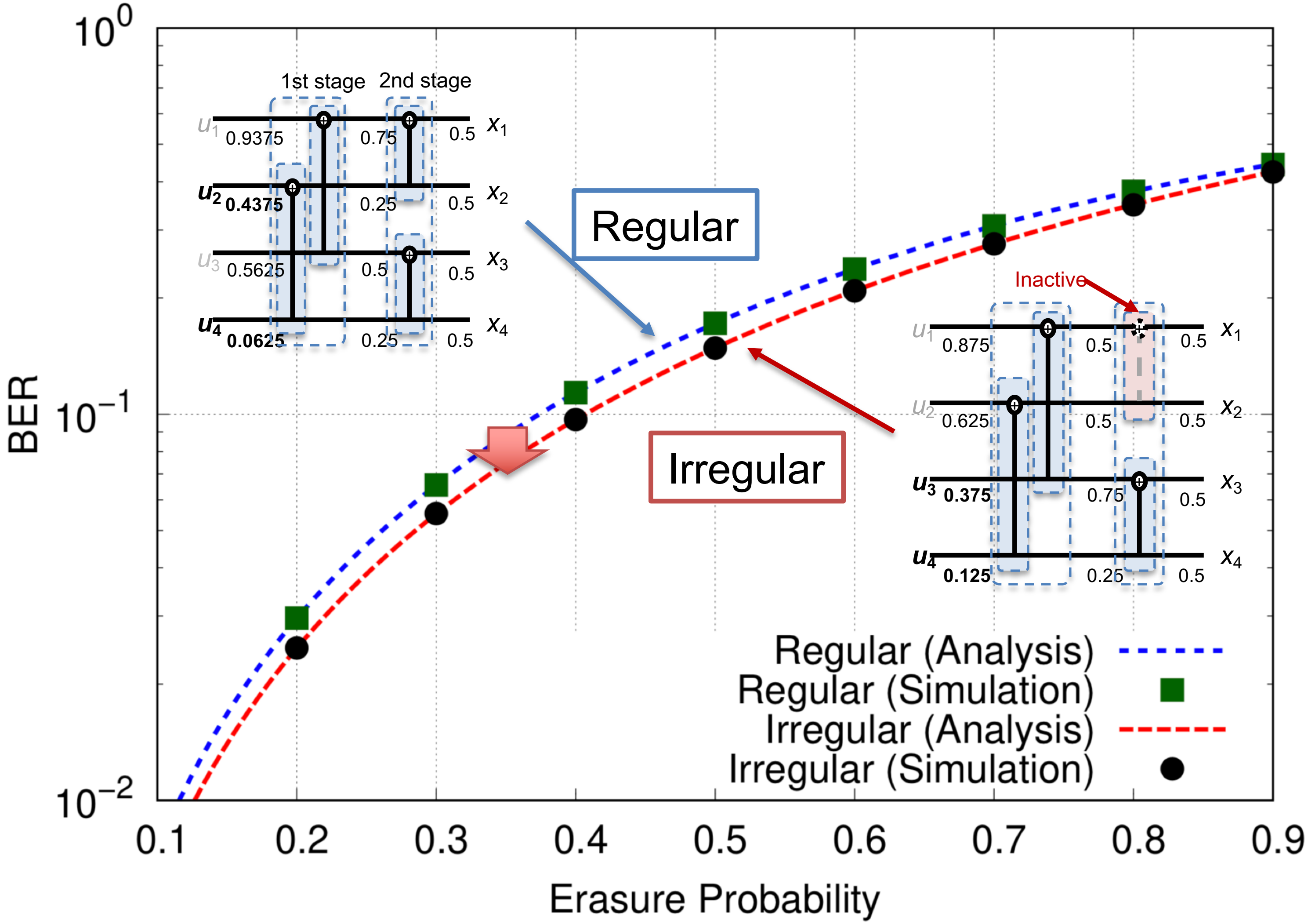


Irregular Polar Code: UB = **0.2383**



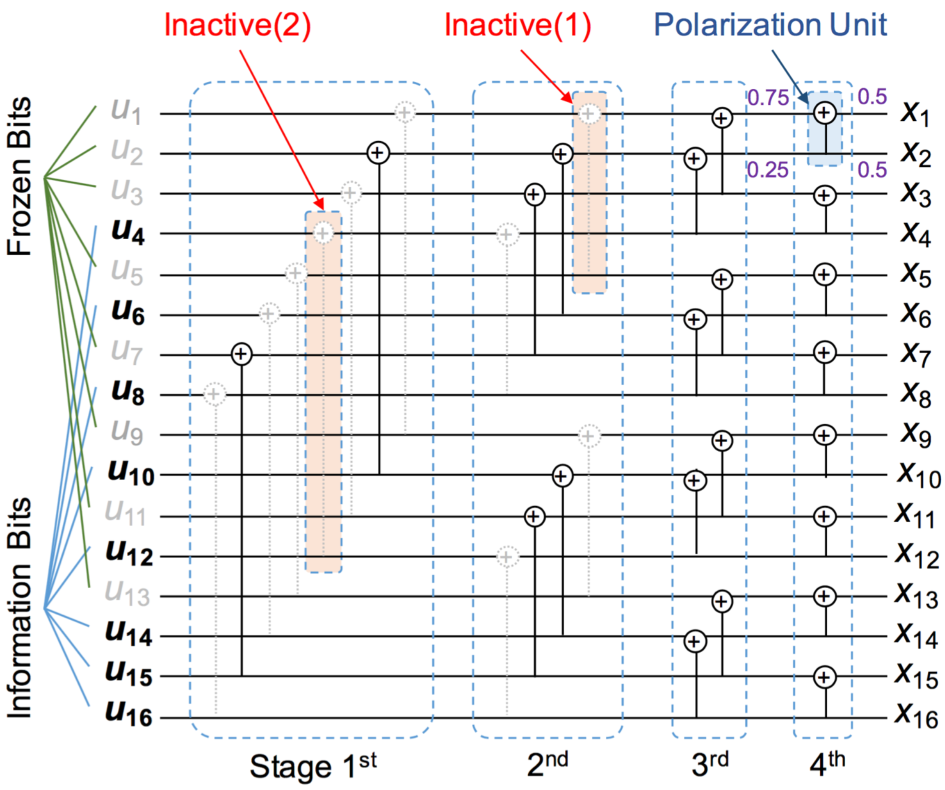
Irregular Polarization Gain

- BER performance in BEC channels; better BER and lower complexity



Irregular Polar Code Design Method

- We proposed greedy design method using **EXIT chart** analysis to jointly design **frozen** bit locations and **inactive** polarization units [GLOBECOM 2017]



Algorithm 1 Joint interleaver and irregular polar codes design

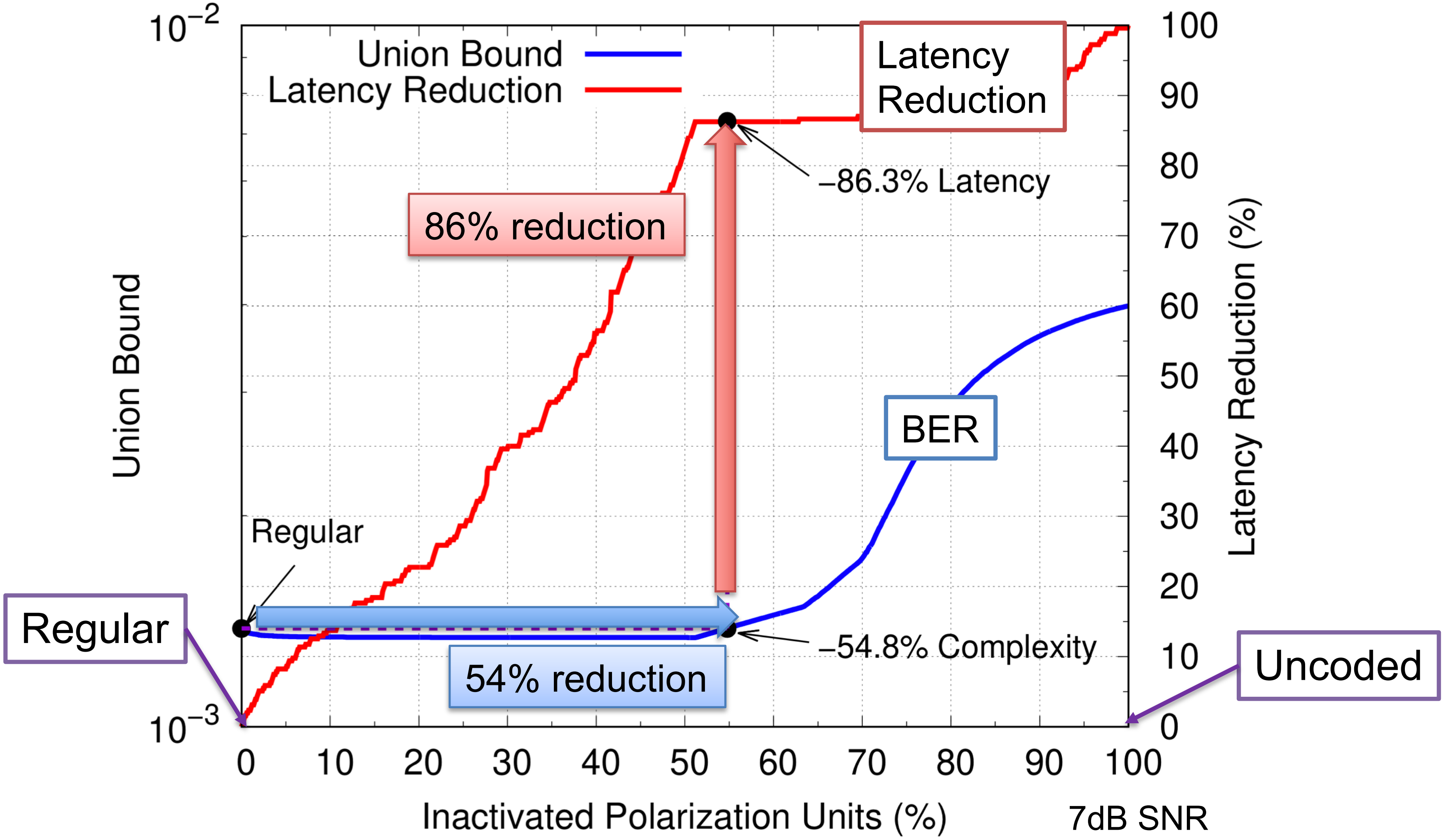
```
Initialize:
  $\tilde{\mathcal{C}} = [\tilde{C}_1, \tilde{C}_2, ..., \tilde{C}_N]$: mutual information of each modulated bit at eigen-mode channels for an ave. SNR of $\rho$
Start:
  for all interleaver sets $\Pi$ in consideration do
    perform de-interleaving: $\mathcal{I} = \Pi^{-1}(\tilde{\mathcal{C}})$
    activate all polarization units
    while $N_{\mathrm{inact}} \in \{1, 2, \ldots, N_{\mathrm{U}}\}$ do
      for all active polarization units do
        inactivate the target polarization unit
        $\mathcal{I}' = \mathrm{UpdateMI}(\mathcal{I})$ according to (7)
        select frozen bits $\mathbb{K}$ having the $N - k$ smallest $\mathcal{I}'$
        calculate the upper bound $P_{\mathrm{e}}$ according to (9)
        reactivate the target polarization unit
      end for
      inactivate the polarization unit having smallest $P_{\mathrm{e}}$
    end while
  end for
  Return: best interleaver, frozen bit locations, and inactivated polarization units achieving the smallest $P_{\mathrm{e}}$
```

$$\mathcal{I}_{r_{\mathrm{U}}}^{[l-1]} = 1 - J\left(\sqrt{\left[J^{-1}(1-\mathcal{I}_{r_{\mathrm{U}}}^{[l]})\right]^2 + \left[J^{-1}(1-\mathcal{I}_{r_{\mathrm{L}}}^{[l]})\right]^2}\right),$$

$$\mathcal{I}_{r_{\mathrm{L}}}^{[l-1]} = J\left(\sqrt{\left[J^{-1}(\mathcal{I}_{r_{\mathrm{U}}}^{[l]})\right]^2 + \left[J^{-1}(\mathcal{I}_{r_{\mathrm{L}}}^{[l]})\right]^2}\right),$$

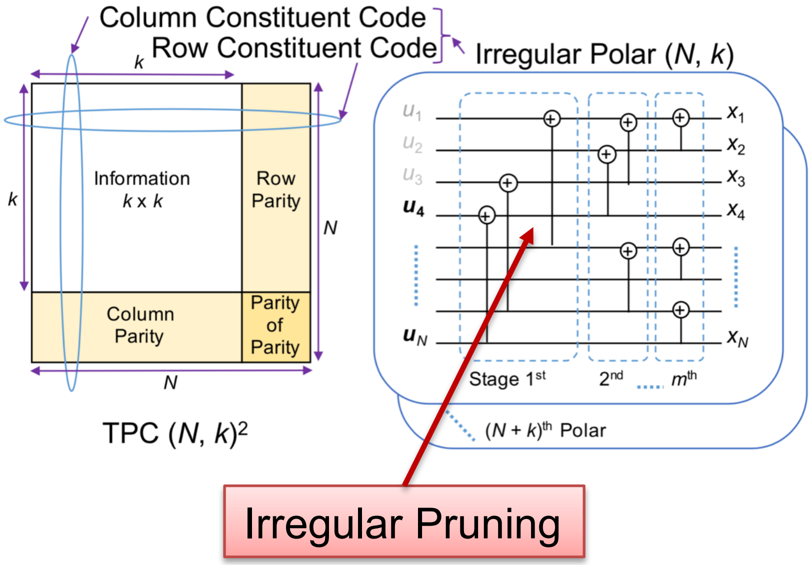
Union Bound, Complexity, and Latency Analysis

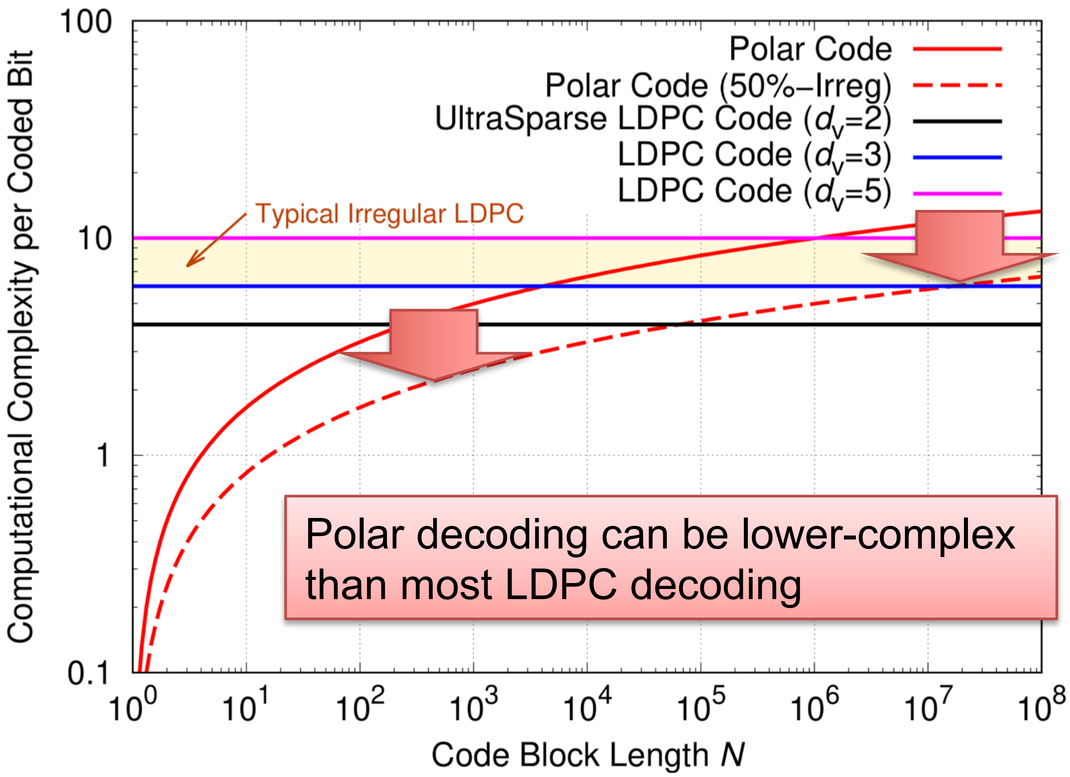
- Polar (256, 239): Lower complexity, lower latency, & improved BER



TPC with Irregular Polar Codes

- Parallel SCL decoding:
  - 256-times higher throughput
- Irregular pruning:
  - 50~70% complexity reduction
  - 80~90% decoding latency reduction
    - Total 256 x 10 = **2560 times** faster decoding

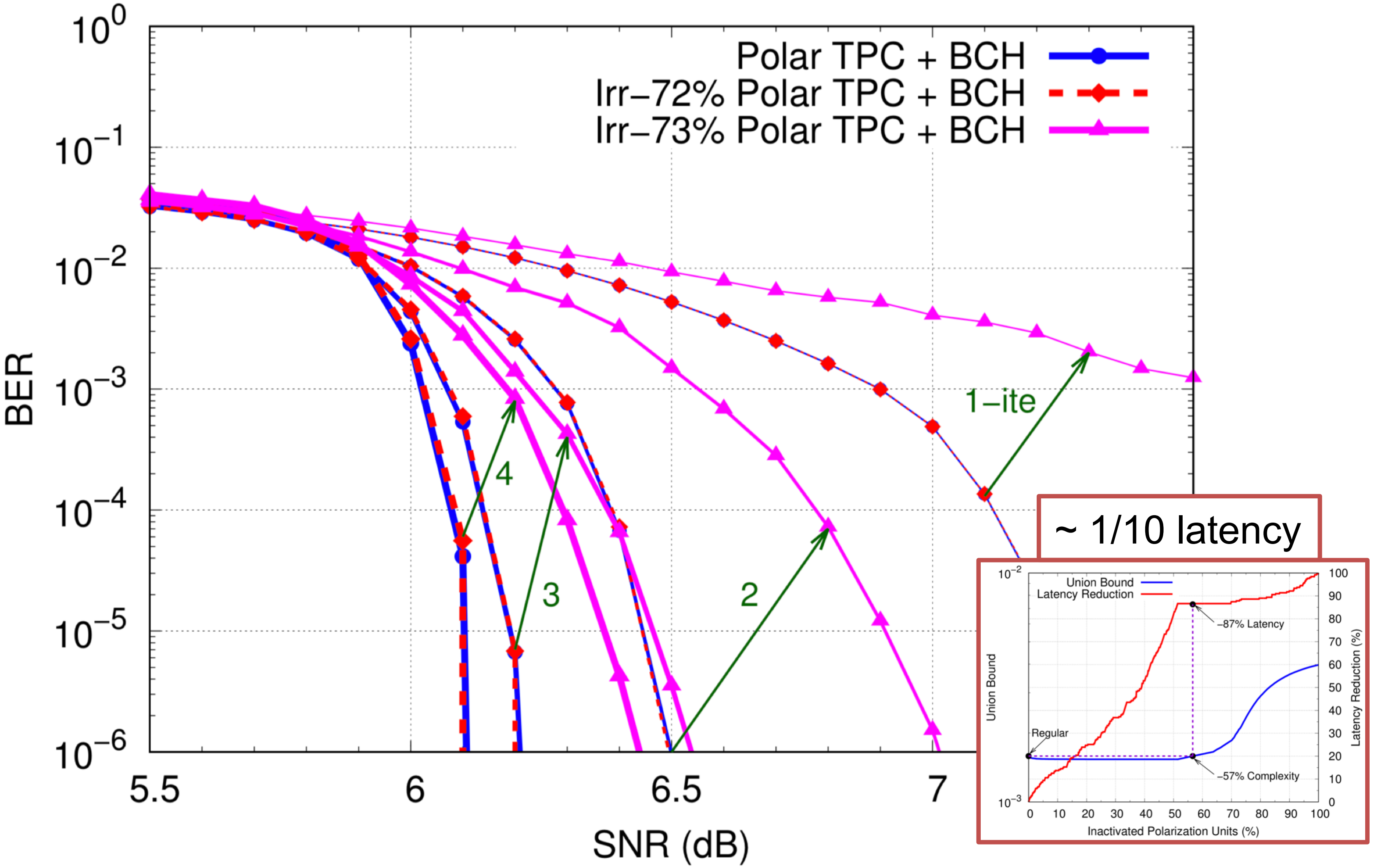




Polar decoding can be lower-complex than most LDPC decoding

Irregular Polar-TPC Performance

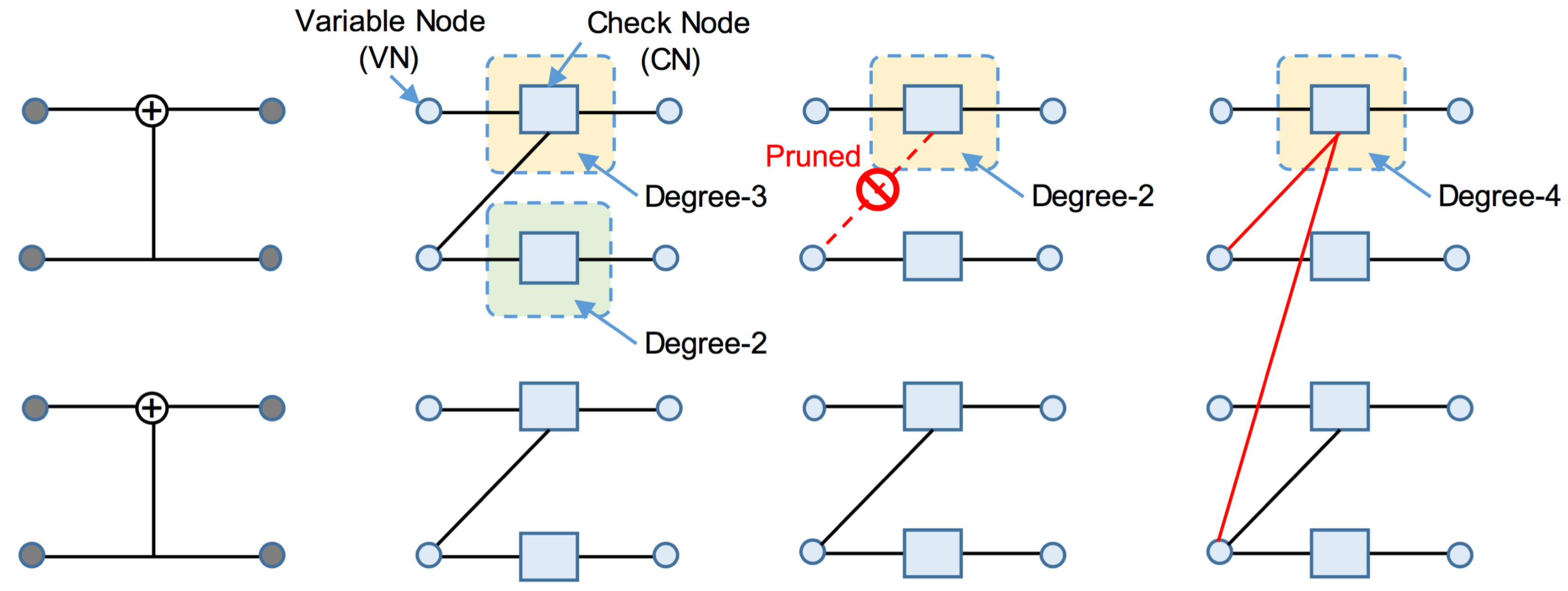
- No performance loss with beyond 50% pruning, up to **72%** reduction



~ 1/10 latency

Side Comment: Irregularity

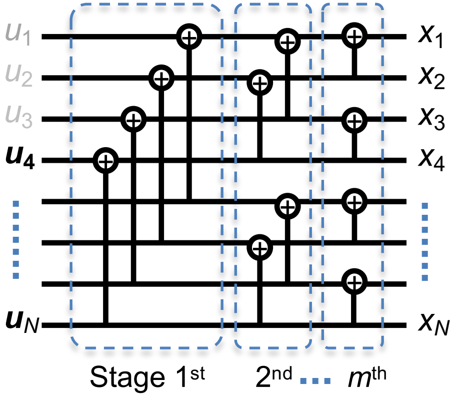
- We can of course consider many other irregular structures not only pruning
  - Grafting edges, mixed multi-kernel, edge looping, etc.
- Irregular polar codes: **There are a lot of rooms to investigate this new family!**

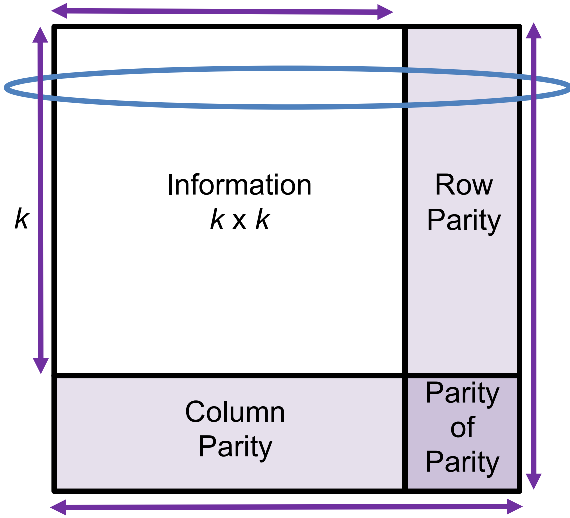


(a) XOR Polar Stage (b) Factor Graph (c) Irregular Pruning (d) Irregular Grafting

Outline

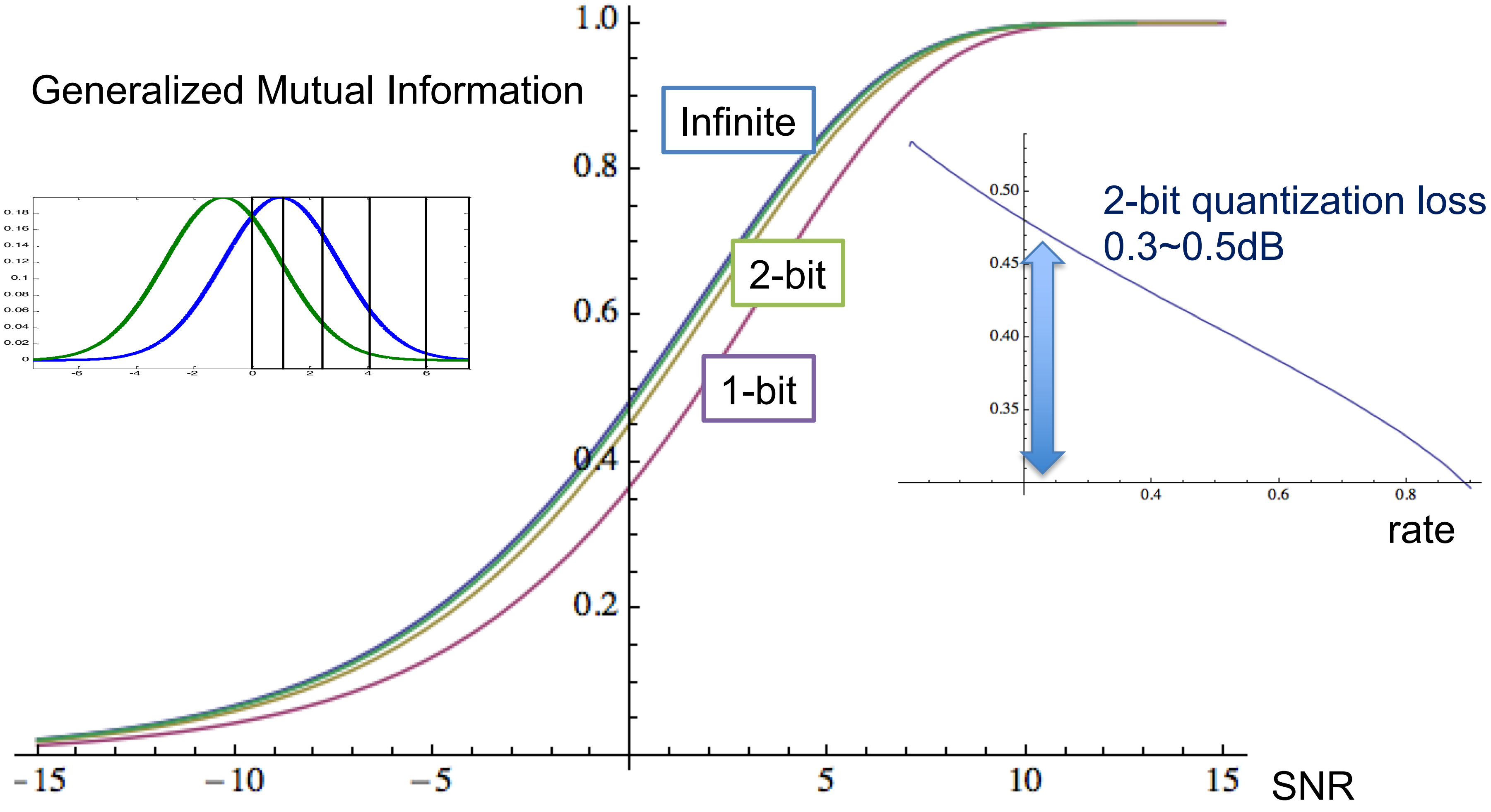
- Polar Coding
  - Successive cancellation list (SCL) decoding + cyclic-redundancy check (CRC)
  - Polar codes vs. low-density parity-check (LDPC) codes
- EXIT chart design method
  - Interleaver design for quadrature-amplitude modulation (QAM)
  - Non-uniform shaped QAM
- Polar-based turbo product codes (TPC)
  - Highly-parallel and pipelining processing
  - SCL-based soft-in soft-output decoding
- Irregular polar coding
  - Pruning polarization units
  - Complexity & latency reduction
- Quantized polar decoding
  - Hardware-friendly operation
  - Look-up decoding optimization
- Summary





Finite Precision Operation

- Hardware implementation typically uses **finite-precision fixed-point** operations
- Quantizing LLR values can limit GMI

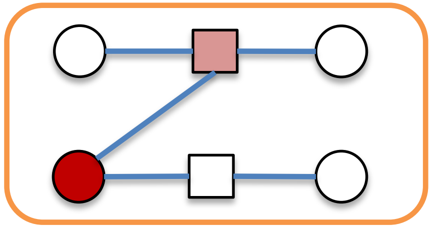

Generalized Mutual Information

2-bit quantization loss 0.3~0.5dB

Quantized SCL Decoding

- Message passing for quantized LLRs can exponentially expand its cardinality

2-in 1-out check-node decoder (CND) → Product



$$L_i' = L_i \boxplus L_j = 2\tanh^{-1}\left(\tanh\frac{L_i}{2} \times \tanh\frac{L_j}{2}\right)$$

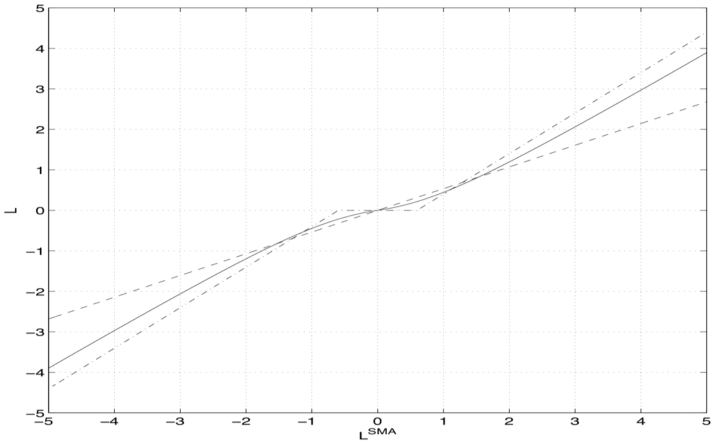
Q-level -> Q²-level

2-in 1-out variable-node decoder (VND) → Sum

$$L_j' = (-1)^{u_i} L_i + L_j$$

Q-level -> 2Q-level

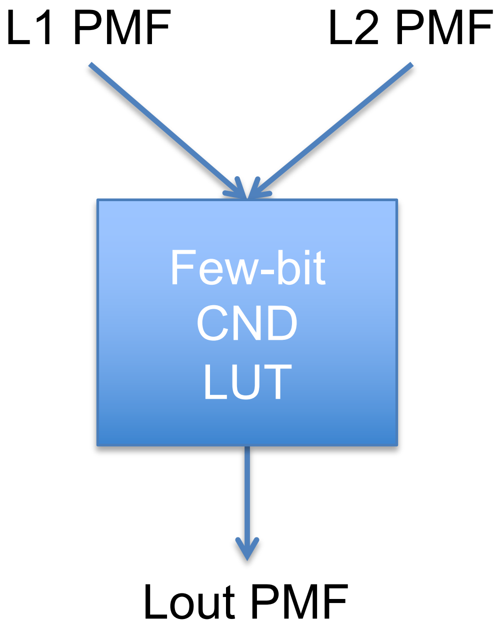
- Nonlinear CND operation often uses
  - **Min-sum**
  - Offset min-sum
  - Delta-min
  - **Look-up table (LUT)**

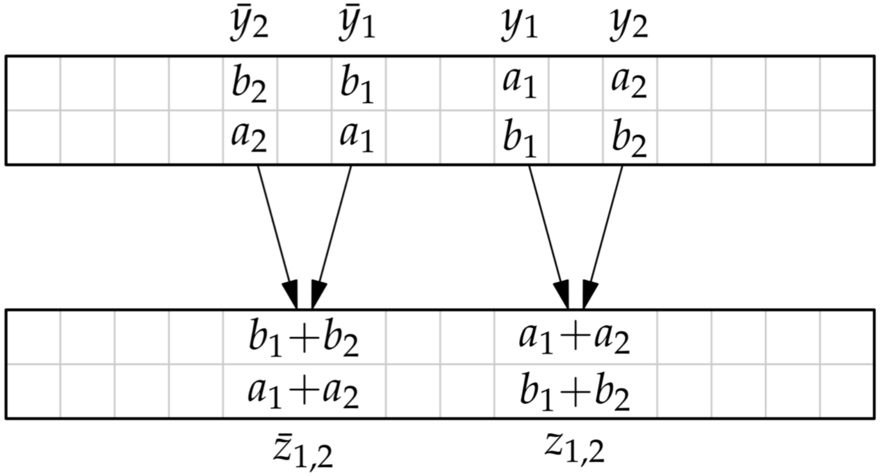


How to Optimize Look-Up Table (LUT)

- Minimizing operation errors is not actually optimal for few-bit LUTs
- We optimize LUT such that the **mutual information is maximized**
  - LUT controls output probability mass function (PMF) directly
  - Analogous to Tal-Vardy's density evolution for encoding optimization
  - We modify the method for decoding optimization with non-uniform quantization

Progressive merging to control the cardinality



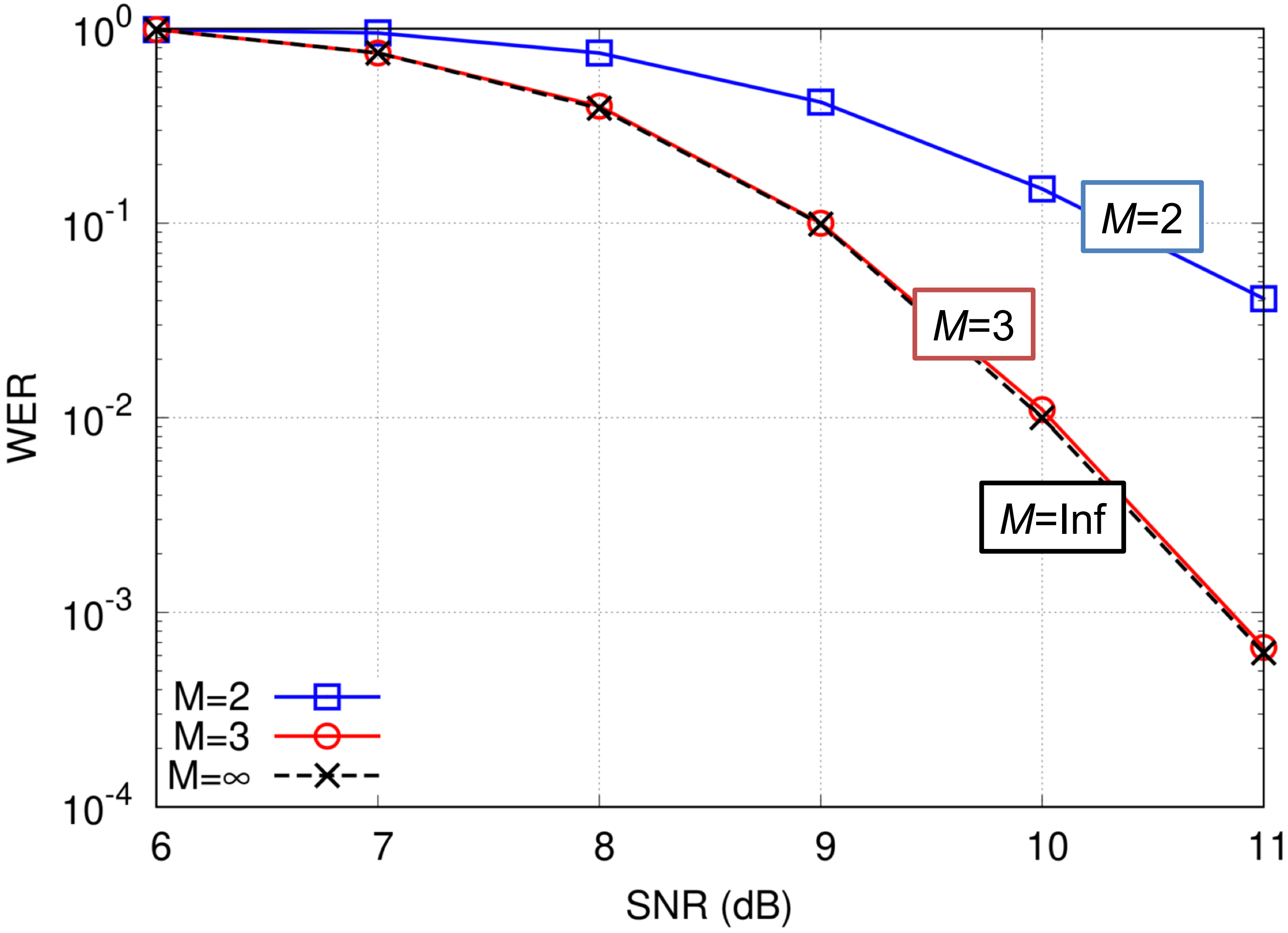


$$C(a,b) = -(a+b)\log_2((a+b)/2) + a\log_2(a) + b\log_2(b)$$

Discrete memoryless channel model

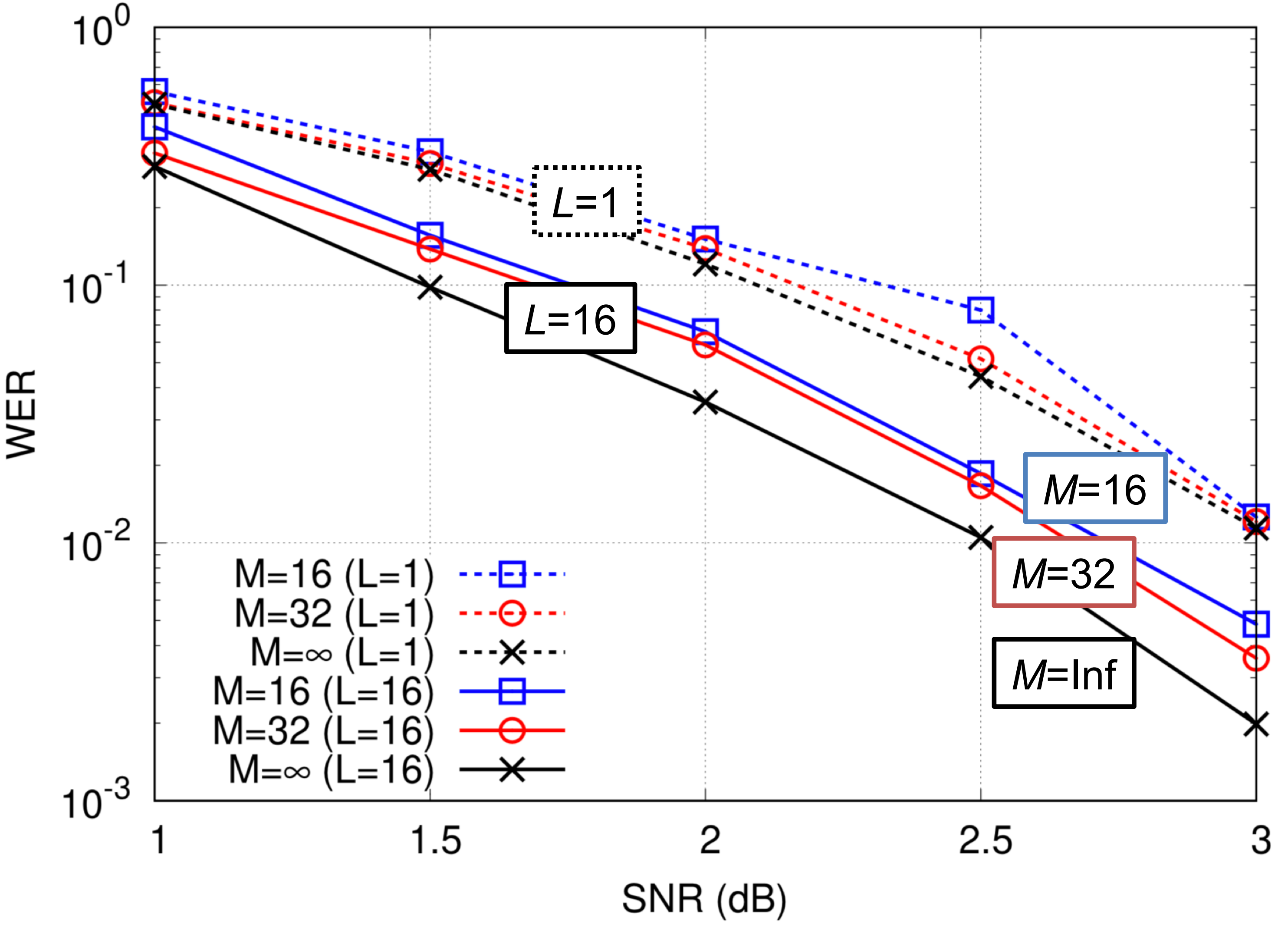
Performance of QSC Polar Decoding (256, 240)

- Ternary amplitude quantization has no loss over floating point decoding



Performance of QSCL Polar Decoding (256, 128)

- More quantization is required for lower rates and more list sizes



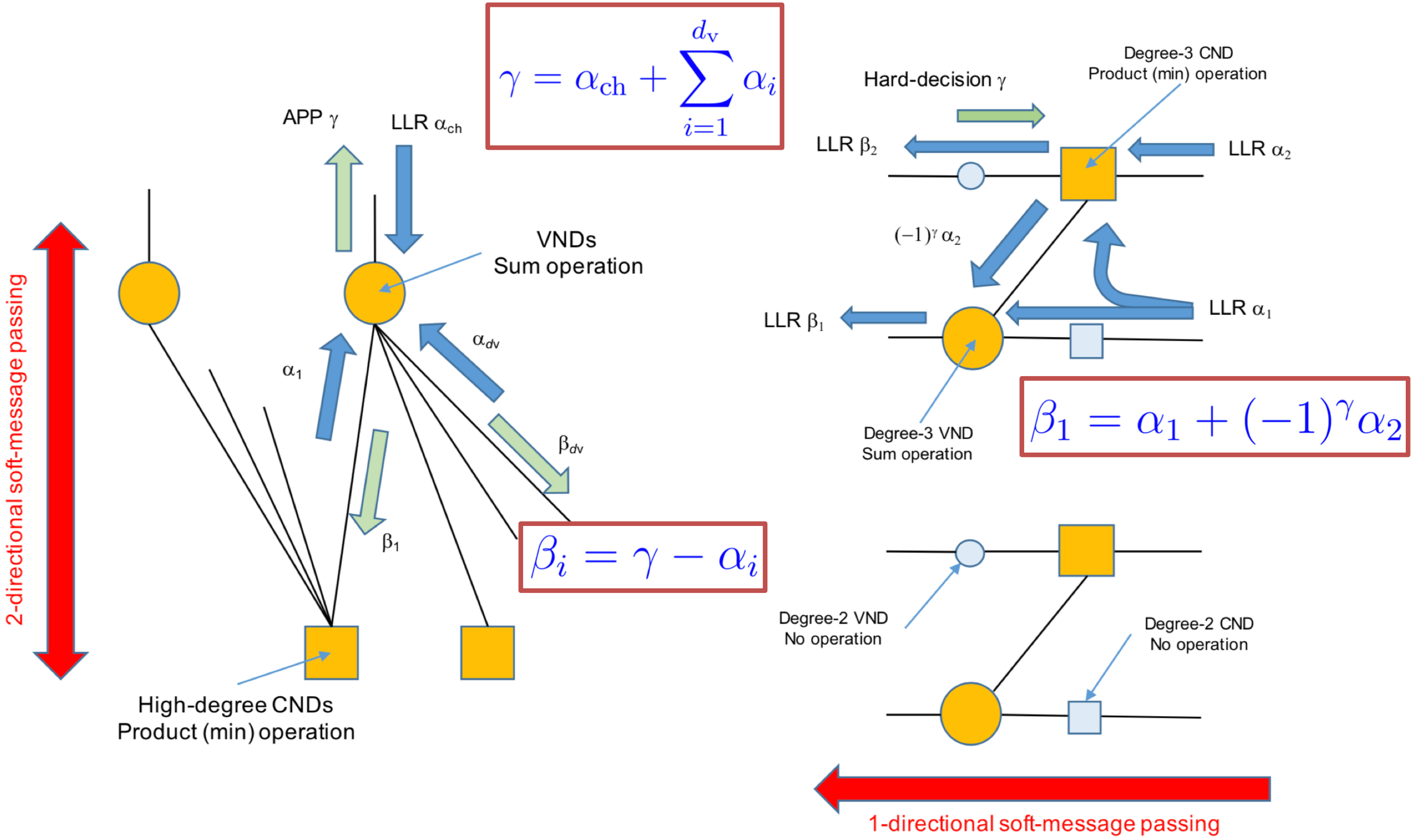
Summary

- We investigated **hardware-efficient polar codes** for high-speed communications
- We briefly introduced recent advancement of polar coding, competitive to LDPC
  - **SCL+CRC decoding**
- We evaluated joint interleaver and frozen design for BICM
  - Interleaver design for shaped QAM achieves **0.9dB gain**
- We proposed polar-TPC to achieve **256-times faster** decoding throughput
  - **0.5dB** better than conventional BCH-TPC
  - **0.2dB** from optimal performance towards long polar+CRC code
- We further reduced complexity and latency by using **irregular polar codes**
  - **72%** complexity reduction
  - **87%** latency reduction: 2500-times faster decoding with TPC
- We introduced **LUT decoding** without any arithmetic operations for reducing hardware complexity furthermore

MERL, Cambridge, Massachusetts, USA

Decoding Complexity

- LDPC BP decoding vs. polar SCL decoding



$$\boxed{\gamma = \alpha_{\mathrm{ch}} + \sum_{i=1}^{d_{\mathrm{v}}} \alpha_i}$$

$$\boxed{\beta_i = \gamma - \alpha_i}$$

$$\boxed{\beta_1 = \alpha_1 + (-1)^{\gamma} \alpha_2}$$

(a) LDPC Codes

(b) Polar Codes

Pyndiah Chase Algorithm with SCL Decoding

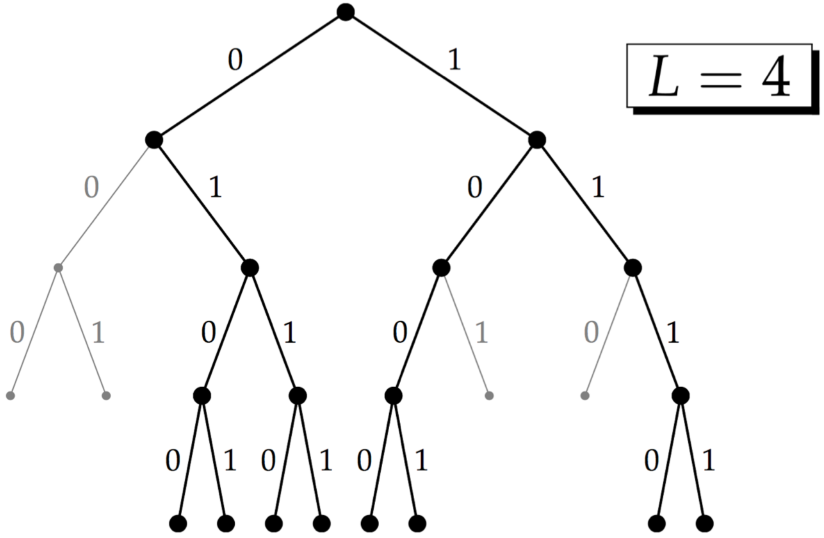
- Chase approximation based on max-log MAP over surviving lists

$$D_\ell = \sum_{j=1}^{N} (y_j - \hat{u}_j)^2$$

$$L_{\mathrm{app}}(i) = \frac{S_{\mathrm{zero}}(i) - S_{\mathrm{one}}(i)}{4}$$ if there is competing pattern

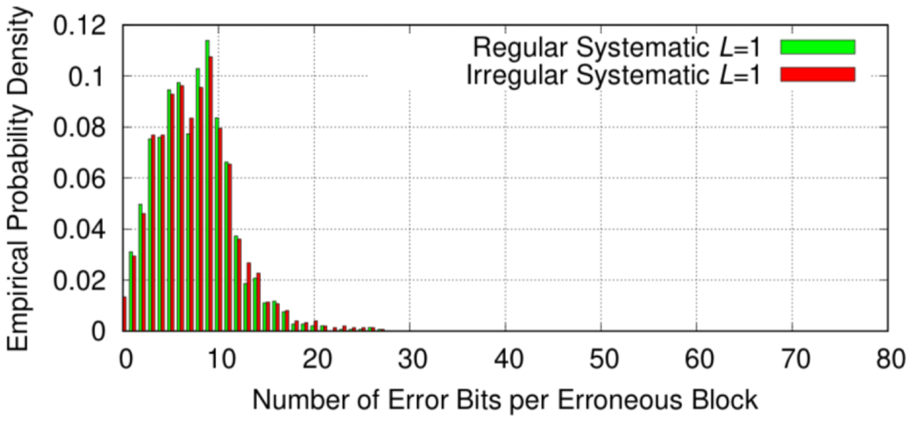
$$L_{\mathrm{app}}(i) = \beta \cdot (2\hat{u}_i - 1)$$ otherwise

$$\alpha \cdot L_{\mathrm{app}}$$

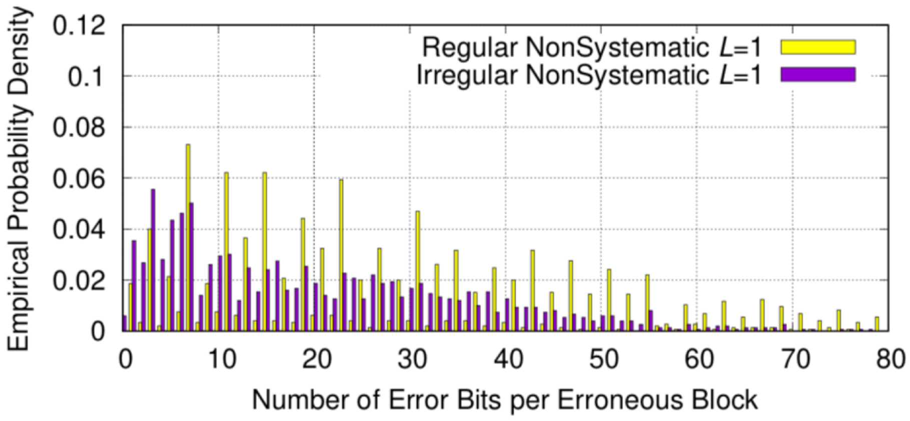


Burst Error Analysis

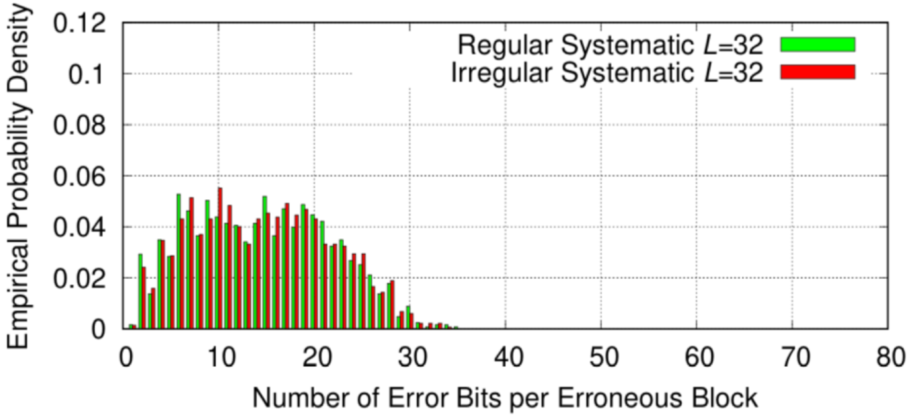
- (256, 240) polar: number of bit errors in erroneous block



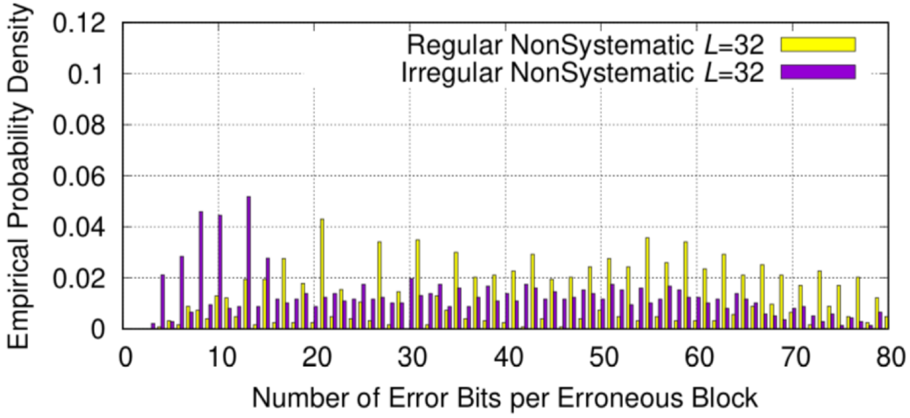
(a) Systematic $L = 1$ (5 dB)



(b) Non-Systematic $L = 1$ (5 dB)



(c) Systematic $L = 32$ (3.5 dB)



(d) Non-Systematic $L = 32$ (3.5 dB)