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Steady-State Analysis of HVAC Performance using Indoor Fans in Control Design

Joaquin G. Ordonez, Claus Danielson, Daniel Limon, Scott A. Bortoff and Stefano Di Cairano

***Abstract*— Indoor fans are high-authority actuators in heating, ventilation, and air conditioning (HVAC) systems since they facilitate the transfer of heat between refrigerant and room air. In some variable refrigerant flow (VRF) systems, the indoor fan speeds are under the control of the occupants, rather than the HVAC control system. This paper studies the benefits of transferring control of the indoor fans to the HVAC controller. We quantify the system performance using five metrics related to occupant comfort and power consumption. The first metric measures the ability of the HVAC system to accommodate users with different temperature preferences by quantifying the largest difference in requested room temperatures that can be achieved with and without the aid of indoor fans. The second and third metrics measure the ability of the HVAC system to reject extreme heating and cooling loads. The final two metrics measure the reduction in power consumption obtained by manipulating the indoor fan speeds. Each of these metrics is computed via linear programming for varying numbers of indoor units. Simulation results indicate that the maximum steady-state difference in room temperatures is tripled, and the maximum rejected heating and cooling loads are doubled. Furthermore, power consumption is significantly reduced.**

I. INTRODUCTION

Heating, ventilation and air conditioning (HVAC) systems are widely used in industrial, commercial and residential buildings to improve the comfort and health of occupants by regulating indoor temperatures and maintaining the flow of fresh air. Occupant comfort is difficult to quantify. It varies from individual to individual and it depends on several variables including air temperature, wall temperature, humidity, and the velocity of the air surrounding the person [1], [2]. Thus, some variable refrigerant flow (VRF) systems grant occupants control of indoor fan speed, allowing them to separately specify the air temperature and speed that they find most comfortable. In addition, placing the fan speed under the control of the occupant provides immediate feedback that lets the occupant know that the HVAC system is working.

Unfortunately, this lack of control of the indoor fan speeds can impede the HVAC system from achieving the desired air temperature since the indoor fans strongly affect the transfer of heat between the refrigerant and the room air. This is especially true when the occupants desire very different room temperatures between rooms or when the heating or cooling loads are extreme. In this paper, we analyze the effect of using the indoor unit fans as control variables in the control system, providing evidence that motivates the design of HVAC controllers that optimally manipulate the fans. The analysis is based on experimentally identified models of a production HVAC system. To the best of our knowledge, even though there is some research in the impact of the outdoor unit fan on the system performance [3], there has not been an analysis of the effect of indoor fans, specifically in the case of multiple indoor zones, each with their own indoor unit, but sharing the same outdoor unit.

The objective of this study is to quantify benefits of placing the indoor fans under the control of the HVAC control system. Performance is quantified using five metrics that measure comfort and power consumption. The first metric measures the ability of the HVAC system to accommodate users with different temperature preferences by quantifying the largest difference in requested room temperatures that can be achieved with and without the aid of indoor fans. The second and third metrics measure the ability of the HVAC system to reject extreme heating and cooling loads. The final two metrics measure the reduction in power consumption obtained by manipulating the indoor fan speeds. The metrics are computed via linear programming (LP) for buildings with varying numbers of rooms. We note that we only need to interrupt occupant control of the fans when that occupant has requested a temperature set-point that is highly inconsistent with the requests of other occupants.

Placing the indoor fans under the authority of the HVAC control system has additional benefits in terms of energy efficiency. Studies of energy consumption in developed countries show that up to 40% of the energy is consumed in buildings, of which HVAC systems account for almost 50% of the energy demand [4], [5]. This has motivated research in advanced control strategies for improving energy efficiency, such as model predictive control (MPC) [6], which as been shown to reduce energy consumption up to 30% in some situations compared to classical control methods [7]. We show that the additional degrees of freedom provided by the indoor fans can be used to significantly reduce power consumption.

II. PROBLEM STATEMENT

This section describes the HVAC system and the study focus of this paper.

A. HVAC plant description

This paper considers the control of Variable Flow Refrigerant (VRF) systems operating in heating mode. This system

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features a single outdoor unit, which includes the compressor and evaporator coil, and multiple indoor units, each of which include an Electronically Controlled Expansion Valve EEV$_i$, a condensing coil, and a variable speed fan, as shown in Fig. 1 [8]. In this configuration, the system acts as a heat pump, moving heat from the colder outdoor air to the warmer indoor air.

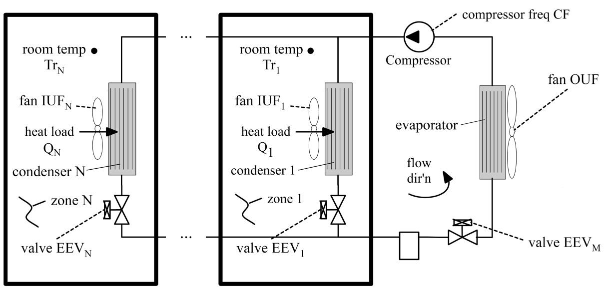

Fig. 1. Multiple indoor unit vapor compression system.

The system operates by compressing gas-phase refrigerant, which is distributed to the indoor units where it condenses inside each of the indoor coils, releasing heat to each indoor zone. The condensed refrigerant, which is usually sub-cooled, then expands as it passes through each of the EEV$_i$. It returns to the outdoor unit as cold two-phase fluid, where it expands a second time as it passes through the EEV$_M$. The cold low-pressure refrigerant then evaporates in the outdoor coil, absorbing heat from the outside air, and returns to the compressor as superheated gas. Conventionally, the actuators under control are the compressor frequency CF, the EEV$_i$, and EEV$_M$ settings, and the outdoor fan speed OUF. The indoor fan speeds IUF$_i$ is not under the authority of the controller, but is set by the user, and can be considered as a measured disturbance from a control perspective.

A relevant feature of this system is that the indoor units are all at the same pressure, neglecting pressure losses in the piping network, which means that the refrigerant condenses at the same temperature in all of the indoor units. The heat flow into each zone can be modulated somewhat using the EEV$_i$, but the range of attainable heat flux is limited, meaning the control authority of each EEV$_i$ is limited. In the steady-state, the heat load disturbance in each zone, Q_i, must equal the heat flux from the corresponding indoor unit. Therefore, the range of heat load disturbances that can be rejected, or the range of set-point temperatures that can be achieved in the different zones, is limited. This fact motivates our interest in using the indoor unit fan speeds IUF$_i$ as additional controlled actuators.

We define two types of outputs (tracked and constrained) and two types of inputs (controls and disturbances), shown in Table I. These sets depend on the number of rooms n_r, with each room containing one indoor unit. The tracked outputs are the room temperatures Tr$_i$.

The system is then modeled by a discrete-time linear time invariant (LTI) state space model

TABLE I
SYSTEM INPUTS AND OUTPUTS

Types	Description	Signals
$y_r \in \mathbb{R}^{n_r}$	Tracked outputs	Tr$_i$
$y_c \in \mathbb{R}^{n_r+3}$	Constrained outputs	Tcomp, Tc, Tdsh, Tsub$_i$
$u \in \mathbb{R}^{2n_r+3}$	Control actions	CF, EEVM, OUF, EEV$_i$, IUF$_i$
$q \in \mathbb{R}^{n_r}$	Disturbances	Q$_i$

$$x^+ = A\,x + B_u\,u + B_q\,q \tag{1a}$$
$$y_r = C_r\,x + D_{ru}\,u + D_{rq}\,q \tag{1b}$$
$$y_c = C_c\,x + D_{cu}\,u + D_{cq}\,q \tag{1c}$$

where the dimension n_x of the non-physical states $x \in \mathbb{R}^{n_x}$ depends on the number of rooms n_r.

The inputs are also subject to constraints such that $u_{min} \leq u \leq u_{max}$ due to the physical limitations of the devices. The constraints define the following admissible regions

$$u \in \mathcal{U} \tag{2a}$$
$$y_c \in \mathcal{Y} \tag{2b}$$

where $\mathcal{U} \in \mathbb{R}^{2n_r+3}$ and $\mathcal{Y} \in \mathbb{R}^{n_r+3}$ are polytopes. The constrained outputs are: compressor discharge temperature (Tcomp < 100 °C), condensing temperature (Tc < 60 °C), discharge superheat temperature (Tdsh = Tcomp $-$ Tc > 10 °C), and subcooling temperature (Tsub$_i$ ≥ 0.5 °C) defined as the difference between the condensing temperature (Tc) and the temperature at the exit of each condenser.

B. Operation of indoor fans and HVAC power consumption

Control engineering is required to properly manipulate actuators (input variables) to achieve the desired closed-loop requirements, represented in terms of tracked output variables. Room temperature is a tracked output of the system. However, the indoor unit fan speed (IUF) is an input of a system actuator. If the user has control on the fans, we can describe a constrained input set $u \in \mathcal{U}_{man}$ such that $u_{IUF} = u_{user}$, where u_{IUF} is the vector of inputs of the indoor fans, which is a subset of the input vector u, and $u_{user} \in \mathbb{R}^{n_r}$ are the manually user-defined fan speeds in every room. The controller is forced to use the fan speeds given by the user.

For the analysis of this paper, we will also define the constrained input set $u \in \mathcal{U}_{auto}$ such that $u_{IUF,min} \leq u_{IUF} \leq u_{IUF,max}$, where the fans can be manipulated by the controller inside the range of the minimum and maximum allowed speeds.

A function that calculates power consumption depends on the manipulable inputs of the plant as well as the state of the refrigerant, and is described as $P(x, u)$. The compressor is the machine that consumes most of the power in the system to increase the enthalpy of the refrigerant, and the consumption of the fans can often be negligible. However, HVAC dynamics are highly coupled [9] and proper use of the fans for efficient heat distribution can lead to less work in the compressor. Therefore, manipulation of the input variables will determine both indoor comfort and power consumption.

III. Performance metrics

In this section we define five performance metrics to quantify the benefits of manipulating the indoor unit fans in the controller.

A. Steady-state reachable room temperatures

Temperature tracking is the ability to use system actuators to drive room temperature to the desired reference. A larger set of reachable room temperatures will result in more real scenarios where it will be possible to achieve user-defined comfortable temperatures.

In steady state, the outputs are

$$y_r^\infty = G_{ru}u + G_{rq}q \tag{3a}$$

$$y_c^\infty = G_{cu}u + G_{cq}q \tag{3b}$$

where $G_{ij} = C_i\,(I - A)^{-1}\,B_j + D_{ij}$ are the steady-state gains, with $i = \{r, c\}$ and $j = \{u, q\}$.

Then, the reachable set of temperatures is defined as

$$R_\infty = \left\{ y_r^\infty \in \mathbb{R}^{n_r} : \begin{array}{r} u \in \mathcal{U}_i \\ y_c^\infty \in \mathcal{Y} \\ (3) \\ q = 0 \end{array} \right\} \tag{4}$$

where y_r^∞ (3a) are the tracked outputs that are possible to reach in steady state, subject to disturbances at the operating point ($q = 0$), while enforcing input and output constraints (2), (3b).

The set $R_\infty \in \mathbb{R}^{n_r}$ depends on the $\mathcal{U}_i = \{\mathcal{U}_{man}, \mathcal{U}_{auto}\}$ chosen, which gives us two reachable sets of temperatures that can be compared to easily visualize the effect of manipulating the indoor unit fans in the tracking problem. A larger set means more scenarios with satisfied indoor comfort, i.e. no steady-state error in Tr_i. This set has n_r dimensions, which only allows graphic results for models with $n_r = 2, 3$.

To further understand the effect of the fans in a wider range of n_r, we extracted a performance metric to measure the size of the reachable set (4). The metric is

$$\begin{aligned} \min_u \quad & (\min\ \underline{y}_r^\infty - \max\ \overline{y}_r^\infty) && (5a)\\ \text{s.t.} \quad & (3) \\ & \underline{y}_r^\infty \geq y_r^\infty(i) \geq \overline{y}_r^\infty && (5b)\\ & q(j) = 0 \qquad \forall j = 1 \dots n_r && (5c)\\ & u \in \mathcal{U}_i && (5d)\\ & y_c^\infty \in \mathcal{Y} && (5e) \end{aligned}$$

which is the maximum temperature difference that can be achieved between any two rooms in steady state.

B. Steady-state rejected heat disturbances

Heat loads are non-measurable disturbances that directly affect room temperature. We also want to determine all possible disturbances that can be rejected in steady state while maintaining room temperatures at the operating point.

The set of rejected disturbances is defined as

$$Q_\infty = \left\{ q \in \mathbb{R}^{n_r} : \begin{array}{r} u \in \mathcal{U}_i \\ y_c^\infty \in \mathcal{Y} \\ (3) \\ y_r^\infty = 0 \end{array} \right\} \tag{6}$$

where q are the heat loads that are possible to reject while keeping tracked outputs (3a) at the operating point ($y_r^\infty = 0$), and limited to input and output constraints (2), (3b). Positive q is heat generation, while negative q is heat loss.

To measure the size of the rejected set (6) for any n_r, we found two appropriate metrics. The first one is

$$\begin{aligned} \max_u \quad & \max\ \overline{q} && (7a)\\ \text{s.t.} \quad & (3), (5d), (5e) \\ & q \geq \overline{q} && (7b)\\ & y_r^\infty(j) = 0 \qquad \forall j = 1 \dots n_r && (7c) \end{aligned}$$

which is the biggest heat generation that we can reject in one room. The metric is subject to the same restrictions of Q_∞ (6).

The second one is

$$\begin{aligned} \min_u \quad & \min\ \underline{q} && (8a)\\ \text{s.t.} \quad & (3), (5d), (5e), (7c) \\ & q \leq \underline{q} && (8b) \end{aligned}$$

which is the largest heat loss that can be rejected in one room (8), subject to the same restrictions of Q_∞ (6).

Problems (5), (7), (8) are linear programming (LP) problems. These metrics depend on the manipulation of the indoor fans through the $\mathcal{U}_i$ chosen, as well as on the model that also depends on the number of rooms n_r.

C. Steady-state power efficiency

Manipulating the indoor unit fans gives the controller extra degrees of freedom that could be used to satisfy the indoor comfort conditions in a way that reduces the power consumption of the plant. The power consumption is modeled by a function $P(x, u)$ obtained from model data. In steady state, the state and inputs are determined by the tracked outputs y_r^∞ that need to be reached and the disturbances q that have to be rejected. To minimize power consumption in steady state, we propose the optimization problem

$$\begin{aligned} P^*(y_r^\infty, q) = \quad \min_u \quad & P(x, u) && (9a)\\ \text{s.t.} \quad & (3), (5d), (5e) \\ & x = Ax + B_u u + B_q q\ . && (9b) \end{aligned}$$

Choosing $\mathcal{U}_{man}$ or $\mathcal{U}_{auto}$, gives us P^*_{man} and P^*_{auto}. Then, the difference is the power savings PS^*, that can be expressed in percentage. To study the power savings for any n_r, we define two performance metrics

$$\%PS_R^* = \frac{\int_{y_r^\infty \in R_\infty} [P_1^*(y_r^\infty, 0) - P_2^*(y_r^\infty, 0)]}{\int_{y_r^\infty \in R_\infty} P_1^*(y_r^\infty, 0)} \tag{10}$$

$$\%PS_Q^* = \frac{\int_{q \in Q_\infty} [P_1^*(0,q) - P_2^*(0,q)]}{\int_{q \in Q_\infty} P_1^*(0,q)} \ , \qquad (11)$$

being the average power savings in the reachable set (10) and rejected set (11), respectively.

IV. Steady-State Performance of Indoor Fans

In this section we will show and discuss the results of computing the sets and metrics defined in section III. The computations are performed in the Multi-Parametric Toolbox 3 [10] and YALMIP [11].

The fixed fan speed U_{user} for $\mathcal{U}_{man}$ was set at the maximum speed (upper constraint) to maximize the heat transfer between the refrigerant and the room air.

The model of the HVAC system used in this section is symmetric with respect to the rooms such that the outdoor inputs (compressor, main valve, outdoor fan) have the same effect to every room, and all rooms and indoor units are identical. This symmetry property allows us to easily augment the model to include any number of rooms. We know that in reality there will be non-symmetric coupling effects between the states of adjacent rooms due to heat transfer through walls. However, this effect can be small compared to the coupling given by the heat distribution of the refrigerant if the rooms have good thermal insulation. Model uncertainty can also be compensated later in the control system with a disturbance estimator.

A. Room temperature tracking

In order to provide intuition of the reachable set of temperatures, Fig. 2 is an example of R_∞ (4) with $\mathcal{U}_{man}$ and $\mathcal{U}_{auto}$ in a model with $n_r = 2$. The green set contains the reachable temperatures when the indoor fans are fixed and cannot be manipulated, and the blue set when the fans are manipulated. The sets are symmetric with respect to the main diagonal $y_r^\infty(1) = y_r^\infty(2)$ because the model considered is symmetric with respect to the rooms. The green set is thinner and goes through the main diagonal. The main diagonal represents equal room temperatures, while the orthogonal direction $y_r^\infty(1) = -y_r^\infty(2)$ indicates temperature separation between the rooms. This secondary direction is very short in the green set, and the blue set widens it.

The gains of the outdoor inputs define the long, main diagonal. The gains of the room inputs (room valves, indoor fans) are smaller in comparison, and they limit the set in the orthogonal, secondary direction. Manipulating the indoor unit fans helps reaching a wider temperature difference before hitting the room input constraints. This translates into a larger set of scenarios in which the HVAC system will be able to achieve the desired room temperatures, satisfying the conditions for indoor comfort set by the users.

Figure 3 shows the metric (5) based on the number of rooms n_r in the model. For $n_r = 2$, this metric is the 1-norm of the vector from the room temperatures at the operating point $(0, 0)$, in the direction $(-1, 1)$, to the border of the sets in Fig. 2.

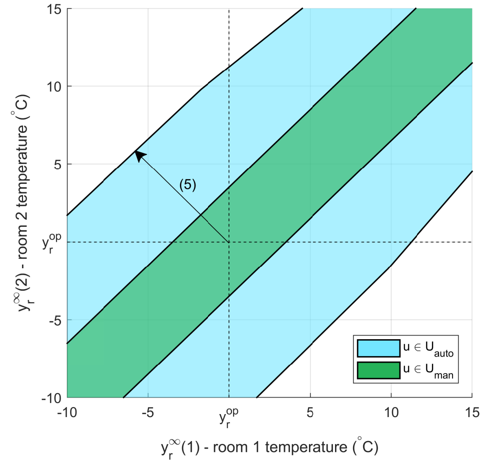

Fig. 2. Reachable set of temperatures for $n_r = 2$.

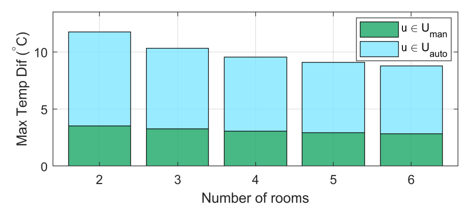

Fig. 3. Max. temperature difference between any 2 rooms, based on n_r.

In terms of reachability, manipulating the indoor fans has a greater benefit with fewer rooms, since room couplings are easier to manage. However, it still provides a significant improvement when the number of rooms is increased, indicating that the behavior seen for $n_r = 2$ in Fig. 2 still applies for $n_r > 2$. Indeed, Fig. 3 shows that the maximum difference in room temperatures almost triples by using the fans and the result is quiet consistent for any n_r.

B. Heat load rejection

Figure 4 shows the set of heat loads that the system can reject (6) with $\mathcal{U}_{man}$ and $\mathcal{U}_{auto}$ in a model with $n_r = 2$. The green set contains the rejected disturbances when the indoor fans cannot be manipulated, and the blue set when they are manipulated. This discussion is similar to the previous one – the sets are more similar in the direction of the main diagonal, and the blue set is wider on the orthogonal direction.

Along the diagonal $q(1) = q(2)$, the sets share two common points. These points are determined by the constraints in the outdoor inputs, which do not change between sets. The peak that appears in the direction where both disturbances are positive (heat generation) represents the system turning off the outdoor actuators to the lower constraint, since the system is in heating mode and has no ability to cool the rooms, except for the natural heat loss due to the nominal disturbances at the operating point. This heat loss is the only factor that compensates heat generation, and manipulating the fans does not change this point. On the side where both disturbances

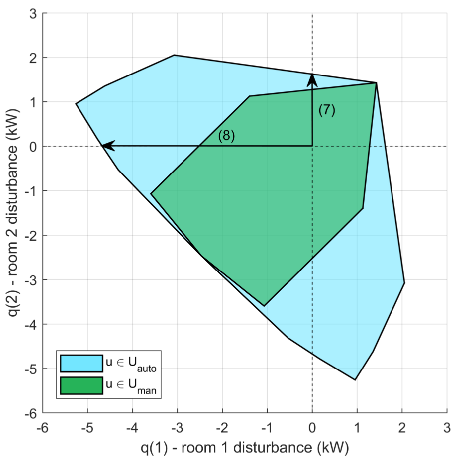

Fig. 4. Rejected set of heat loads for $n_r = 2$.

are negative (heat loss), the limits of the sets have the form of a long diagonal that represents the maximum global heat loss that can be rejected, and is determined by the upper constraints of the outdoor inputs. The other two orthogonal directions, representing heat generation in one room and heat loss in the other one, are determined by the constraints in the room inputs.

Controlling the indoor unit fans provides a larger gain along those directions before hitting the limits of the room valves, broadening the set of rejected heat loads. This also allows the system to maintain comfort conditions under more different scenarios.

Figure 5 shows the metrics (7) and (8) based on the number of rooms n_r in the model. For $n_r = 2$, these metrics are the 1-norm of the vectors from the operating point of heat loads $(0, 0)$, in the directions $(-1, 0)$ and $(1, 0)$, until the limit of the sets, in Fig. 4.

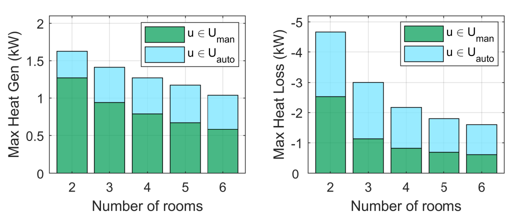

Fig. 5. Maximum rejected heat generation and minimum rejected heat loss in a room, based on n_r.

In heat load rejection, the benefits of manipulating the indoor unit fans are larger when facing heat loss, since the system is operating in heating mode. Figure 5 shows that the maximum rejected heat loss is about twice as much when using the fans.

C. Power savings

Figure 6 shows power savings $PS^* = P_1^* - P_2^*$ in the common reachable set R_∞ with $\mathcal{U}_{man}$, since $\mathcal{U}_{man} \subset \mathcal{U}_{auto}$, for a model with $n_r = 2$. Indeed, power savings cannot be calculated in points that are not reachable without indoor fans. The colors represent computed power savings depending on the point inside the set.

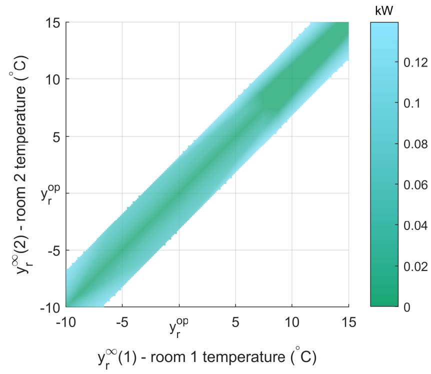

Fig. 6. Power savings in the reachable set of temperatures, for $n_r = 2$.

It is important to note that the power savings map is non-negative, which means that manipulating the fans allows a closed-loop operation that is always more energy efficient. Along the main diagonal, the savings are close to 0 because the optimal solution is to have both indoor fans at the maximum speed On both sides of the elongated shape, the savings become significant because the optimal way to achieve different temperatures in the rooms is to turn down the fan in the room that needs less heat. Controlling the indoor fans allows better distribution of the heat after the compressor, also allowing to reduce its frequency and power consumption.

Figure 7 shows the percentage of power savings (10) in the set where both approaches can reach the desired temperatures and the set where both approaches can reject heat loads, depending on the number of rooms n_r.

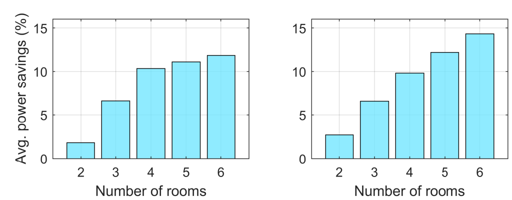

Fig. 7. Power savings in the reachable set of temperatures (left) and the set of rejected heat loads (right), based on n_r.

The percentage of savings is low for 2 rooms, but it quickly becomes significant at 4 rooms, with about 10% of power savings. The savings grow slightly as we add more rooms because scenarios with different room temperature references are more common.

V. Transient Performance of Indoor Fans

We ran closed-loop simulations to evaluate the transient performance while validating the steady state. We used an MPC controller for tracking based on [12]. The controller uses a steady-state target optimizer (SSTO) based on the optimization problem in (9) that calculates a steady state that is admissible by definition and minimizes power consumption.

In this section, the model used is obtained from system identification from data on an experimental setup based on commercial units built at the in Mitsubishi Electric Research Laboratories at Cambridge, MA, USA [8]. Because of that, the model no longer has symmetry on the rooms.

MPC is used for controlling the transients to the steady states. The purpose is to evaluate the transient performance to the reachable temperatures and rejected disturbances that we obtained previously, with the addition that the constraints need to be satisfied during the transient, which was not considered before. Figures 8 and 9 show the simulation results, with the top plots using $\mathcal{U}_{man}$ constraints and the bottom plots using $\mathcal{U}_{auto}$. The controller has the same tuning in both cases.

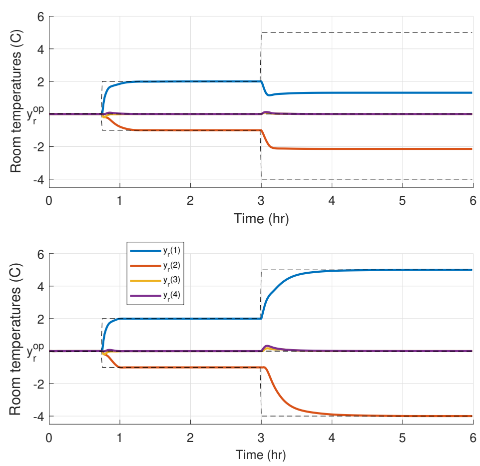

Fig. 8. Simulation with reference changes in rooms 1 and 2.

Figure 8 shows simulation results of temperature reference changes at times around 1 and 3 hours. Looking back at Fig. 3 for $n_r = 4$, the maximum achievable temperature difference is slightly above 3 °C with fixed fan speeds and close to 10 °C with manipulable fan speeds. For the first change at 1 hour, we chose room temperatures such that there is a difference of 3 °C in two rooms. This scenario is possible to track in both cases, backing up the previous results. In the bottom plot, the reference is also achieved faster because using the fans as control inputs provides additional gains. The second reference change requires this difference to be 9 °C, which is unreachable without manipulating the indoor unit fans. In the top plot, the controller achieves a steady state that is as close as possible to the solution, but the temperature difference cannot be higher because the expansion valves EEV_i for those two rooms are saturated. In the bottom plot, the controller achieves tracking and neither EEV_i nor IUF_i are saturated yet, although they are very close to their limits. This also makes the settling time longer.

Figure 9 shows simulation results with two heat loss scenarios in the same room. Around the 1-hour mark, a -600 W heat load is introduced in room 1. This is possible to reject without fans (see Fig. 5). After the 2-hour mark, the heat load introduced changes to -2 kW. This scenario is only possible

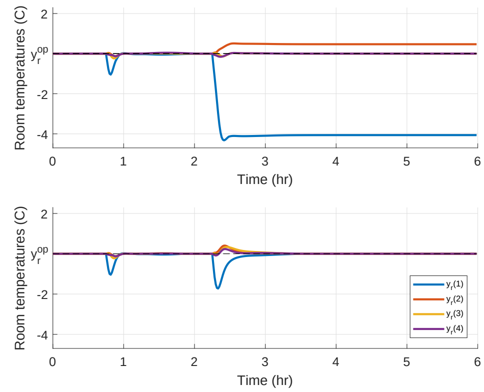

Fig. 9. Simulation with heat loss changes in room 1.

to reject by manipulating the indoor unit fans in the bottom plot since $q \in Q_\infty$ only with $u \in \mathcal{U}_{auto}$.

VI. Acknowledgments

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