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Abstract

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ROBUST SENSOR LOCALIZATION BASED ON EUCLIDEAN DISTANCE MATRIX

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ABSTRACT

In remote sensing systems, exact knowledge of the sensor locations is critical for generating focused images. In order to accurately locate misplaced or perturbed sensors from their received signal data, we proposed a robust sensor localization method based on low-rank Euclidean distance matrix (EDM) reconstruction. To this end, an EDM of sensors and objects is defined and partially initialized by computing distances between the inaccurate sensor locations and distances from the sensors to the objects using signal coherence analysis. We then decompose the noisy EDM with missing entries into a low-rank EDM corresponding to true sensor locations and a sparse matrix of distance errors by solving a constrained optimization problem using the alternating direction method of multipliers (ADMM). We verify our method with simulations on a uniform linear array with unknown perturbations up to several wavelengths.

***Index Terms*—** Sensor localization, Euclidean distance matrix (EDM), auto-focus imaging, sparsity

1. INTRODUCTION

In remote sensing systems, sensor locations play a crucial role in the imaging process. When sensor locations are exactly known, we may generate a well focused image of the region of interest from received data by compensating the phase change of received data relative to the source signal in the inverse imaging process. However, in practice sensor location errors may be several multiples of the source signal wavelength due to misplacements, poor calibrations, or random perturbations, especially for distributed systems or moving platforms such as airborne or vehicle mounted radar systems. These sensor location errors, if not well estimated, typically lead to out-of-focus or even meaningless imaging results. Therefore, it is desirable to accurately locate sensors in order to realize focused imaging.

Auto-focus has been a challenging problem in remote sensing using different sensor modalities. Location or motion compensation based methods seek to compensate sensor location errors such that different location-induced phase errors can be corrected. If the sensor location errors are much smaller than the source wavelength, the location-induced out-of-focus problem may be solved by sparsity-driven auto-focus algorithms, which model the auto-focus imaging problem as an optimization problem with a perturbed projection matrix and a constraint on the sparsity of reconstructed image [1–3]. The resulting solution, however, includes an error that is related to the location mismatch [4,5]. A global optimal solution is only achievable when location errors are much smaller than the central frequency wavelength and with a good initialization. When location errors are greater than the wavelength, this method may not converge to a focused imaging result.

Inspired by the recent work of Euclidean distance matrix (EDM) [6] in indoor radar imaging and that of robust principal component analysis [7, 8], we propose a novel robust location estimation method to conquer the sensor localization problem in autofocus imaging, especially for over-wavelength location errors. To this end, a noisy EDM of sensors and objects under detection is partially initialized as follows. Distances between the sensors are computed using the ideal sensor locations, distances between the sensors and the objects are estimated by analyzing the correlation of received data, and distances between the objects remain unknown since there is no direct information to estimate them. It is clear that distances between sensors are with errors depending on the level of sensor perturbations. While distances between sensors and objects are generally accurate when the signal-to-noise ratio is high. However, we note that in some cases the distance between a sensor to one object may be treated as the distance between the sensor to another object, causing a spike error in the EDM matrix. Our objective here is then to recover the underlying low-rank EDM from this noisy distance matrix with missing entries. Motivated by the work of robust PCA problem with missing data [8], we employ the ADMM method [9] to decompose the noisy EDM into two parts, one is a low rank EDM corresponding to the true sensor and scattering object locations, the other is the sparse part corresponding to the location errors.

2. PROPOSED METHOD BASED ON EDM

2.1. Background

Consider a collection of N points $\{\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N\}$ in the d-dimensional Euclidean space, where $\mathbf{x}_i \in \mathcal{R}^d$. The squared

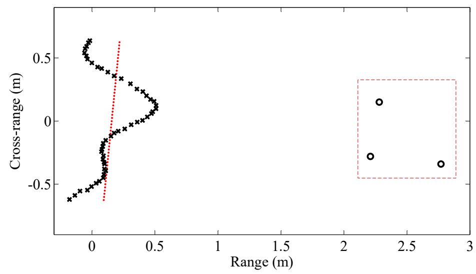

Fig. 1. Setup of random sensor array imaging.

Eulidean distance between $\mathbf{x}_i$ and $\mathbf{x}_j$ is given by

$$\|\mathbf{x}_i - \mathbf{x}_j\|^2 = \mathbf{x}_i^T\mathbf{x}_i - 2\mathbf{x}_i^T\mathbf{x}_j + \mathbf{x}_j^T\mathbf{x}_j. \tag{1}$$

The corresponding Euclidean distance matrix (EDM) of $\{\mathbf{x}_i\}_{i=1,...,N}$ is defined as

$$\begin{aligned}\mathbf{E} =& [\|\mathbf{x}_i - \mathbf{x}_j\|^2] \\ =& \mathbf{1}diag(\mathbf{X}^T\mathbf{X})^T - 2\mathbf{X}^T\mathbf{X} + diag(\mathbf{X}^T\mathbf{X})\mathbf{1}^T,\end{aligned} \tag{2}$$

where $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N] \in \mathcal{R}^{d\times N}$. We can see that in (2), the EDM is the sum of three matrices, which are of rank 1, d, and 1 repectively. Therefore, the rank of the EDM is at most $d+2$. For 2-D or 3-D imaging problems, the rank is at most 4 (for $d = 2$) or 5 (for $d = 3$), respectively, regardless how many sensors are included in $\mathbf{X}$.

It is also straightforward to demonstrate that any translation, rotation, or reflection transform of $\mathbf{X}$ will lead to the same EDM [6]. Therefore, given an EDM, there is no unique solution for the point coordinates. However, given a reference point, a possible solution can be achieved by the following process. Let $\mathbf{x}_1 = 0$ be the origin, and $\mathbf{e}_1$ be the first column of $\mathbf{E}$. The Gram matrix of $\mathbf{X}$ can be computed using EDM as

$$\mathbf{G} = \mathbf{X}^T\mathbf{X} = -\frac{1}{2}(\mathbf{E} - \mathbf{1}\mathbf{e}_1^T - \mathbf{e}_1\mathbf{1}^T). \tag{3}$$

Using the singular value decomposition (SVD)

$$\mathbf{G} = \mathbf{U}^T\mathbf{\Lambda}\mathbf{U}, \tag{4}$$

where

$$\mathbf{\Lambda} = diag(\lambda_1, ..., \lambda_N), \tag{5}$$

with eigenvalues sorted in the order of decreasing magnitude, we can reconstruct a point set $\hat{\mathbf{X}}$ from the original EDM as follows

$$\hat{\mathbf{X}} = [diag(\sqrt{\lambda_1}, ..., \sqrt{\lambda_d}), \mathbf{0}_{d\times(N-d)}]\mathbf{U}^T. \tag{6}$$

2.2. Noisy EDM formation

For simplicity, we consider a 2-D array imaging problem in which a mono-static radar is moving along a predesigned trajectory to detect K static objects situated in a region of interest (ROI), as shown in Fig. 1. We assume the trajectory is a straight line and the radar acts as a virtual uniform linear array of N sensors, represented by the red dots in Fig. 1. In order to accurately locate sensors, we treat each sensor and each object in the ROI as a point of interest to build an EDM of them. The idea is that we first build a noisy and incomplete EDM according to measurements and prior information about the sensors. Then we recover a low-rank EDM based on this noisy EDM. the low-rank EDM is determined, the sensor locations as well as the object locations can be explored accordingly given a reference point.

Let $P(\omega_m)$ be the frequency-domain source signal emitted by the mono-static radar. The signal received by radar sensor located at the n^{th} perturbed location $\widetilde{\boldsymbol{r}}_n$ can be modeled as a superposition of radar echoes of all objects in the area of interest as follows

$$Y(\omega_m, \widetilde{\boldsymbol{r}}_n) = \sum_{k=1}^{K} P(\omega_m)S(\omega_m, \boldsymbol{l}_k)\mathrm{e}^{-\mathrm{j}\omega_m\frac{2\|\widetilde{\boldsymbol{r}}_n - \boldsymbol{l}_k\|}{c}}, \tag{7}$$

where $S(\omega_m, \boldsymbol{l}_k)$ is the equivalent complex-valued impulse response of the object at location $\boldsymbol{l}_k$, and $\mathrm{e}^{-j\omega_m\frac{2\|\widetilde{\boldsymbol{r}}_n - \boldsymbol{l}_k\|}{c}}$ presents the phase change of received radar echo relative to the radar source signal after propagating a distance of $2\|\widetilde{\boldsymbol{r}}_n - \boldsymbol{l}_k\|$ at speed c.

Let $\mathbf{\Phi} \in \mathcal{R}^{(N+K)\times(N+K)}$ be the noisy EDM of sensors and objects. We estimate its entries using the following process. First, according to (7) , the phase change of received echo relative to the transmitted source signal reflects the distance between the sensor and the corresponding object. For a wide-band source signal, the phase change is characterized by a time shift between the source signal and received signal. This time shift can be effectively estimated by computing the cross-correlation (CC) between the two time-domain signals or computed equivalently in the frequency domain. The maximum of the cross-correlation function indicates the point in time where the signals are best aligned, *i.e.*,

$$\begin{aligned}\widetilde{\tau}_{n,k} &= \mathrm{argmax}_\tau \int y(t, \widetilde{\boldsymbol{r}}_n, \boldsymbol{l}_k)\cdot p(t+\tau)dt \\ &= \mathrm{argmax}_\tau \mathcal{F}^{-1}\{(\widetilde{\mathbf{y}}_{nk}(-j\omega t))^* \odot \mathbf{p}(-j\omega(t+\tau))\},\end{aligned} \tag{8}$$

where $\odot$ represents element-wised product. The $(n, N+k)^{th}$ entry of $\mathbf{\Phi}$ is estimated by

$$\phi_{n,N+k} = \left(\frac{\widetilde{\tau}_{n,k}\cdot c}{2}\right)^2. \tag{9}$$

Second, the Euclidean distance between sensors can be estimated using the ideal sensor locations $\{\boldsymbol{r}_n\}$ as

$$\phi_{n_1,n_2} = \|\boldsymbol{r}_{n_1} - \boldsymbol{r}_{n_2}\|^2, \ \text{for } 0 < n_1, n_2 < N. \tag{10}$$

Since we have no clear information about the object locations so far, the corresponding distances remain unknown, meaning the estimated EDM is only partially observed. In addition, as (8) is generally not concave, multiple local maxima may exist, potentially causing spike errors in the estimated EDM. All these issues will be tackled in our robust sensor localization method.

2.3. Robust EDM analysis

The robust principal component analysis (RPCA) method [7] aims to solve the following problem. Given $\mathbf{\Phi} = \mathbf{E} + \mathbf{S}$, where $\mathbf{E}$ and $\mathbf{S}$ are unknown, but $\mathbf{E}$ is known to be low rank and $\mathbf{S}$ is known to be sparse, recover $\mathbf{E}$ and $\mathbf{S}$. In our case, $\mathbf{\Phi}$ is partially estimated using measurements and prior information of sensor locations. Our objective is then to recover the underlying low-rank $\mathbf{E}$, which is defined as

$$\mathbf{E} = [\|\boldsymbol{x}_i - \boldsymbol{x}_j\|^2], \tag{11}$$

where $\forall \boldsymbol{x}_i, \boldsymbol{x}_j \in \{\widetilde{\boldsymbol{r}}_1, ..., \widetilde{\boldsymbol{r}}_N, \boldsymbol{l}_1, ..., \boldsymbol{l}_K\}$.

To solve this problem, we aim to minimize a constrained cost function for robust sensor localization based on EDM

$$\begin{aligned} \min_{\{\widetilde{\boldsymbol{r}}_n\},\{\boldsymbol{l}_k\}} f = \frac{\gamma}{2}\|(\mathbf{\Phi}-\mathbf{E}-\mathbf{S}) \odot \mathbf{M}\|_F^2 + \\ |vec\{\mathbf{S} \odot \mathbf{M}\}|_1 + \frac{\sigma}{2}\sum_n \|\widetilde{\boldsymbol{r}}_n - \boldsymbol{r}_n\|^2, \\ s.t. \quad \|\widetilde{\boldsymbol{r}}_n - \widetilde{\boldsymbol{r}}_{n+1}\| < \epsilon(\lambda), \\ \frac{1}{N}\sum_{n=1}^{N}\widetilde{\boldsymbol{r}}_n = \frac{1}{N}\sum_{n=1}^{N}\boldsymbol{r}_n, \\ \sum_{n=1}^{N}(\widetilde{\boldsymbol{r}}_n - \bar{\widetilde{\boldsymbol{r}}}) \times (\boldsymbol{r}_n - \bar{\boldsymbol{r}}) = 0, \end{aligned} \tag{12}$$

where $\mathbf{M}$ is a binary matrix corresponding to the partial observed entries in $\mathbf{\Phi}$, in particular, with ones corresponding to the distances between sensors and objects, and zeros elsewhere; $\bar{\widetilde{\boldsymbol{r}}}$ and $\bar{\boldsymbol{r}}$ represent the centers of perturbed and ideal sensor locations, respectively. The 2-D vector cross product is defined as follows

$$\boldsymbol{r}_1 \times \boldsymbol{r}_2 = (x_1, y_1) \times (x_2, y_2) \triangleq x_1 y_2 - x_2 y_1. \tag{13}$$

In (12), we seek a robust solution of sensor locations by imposing sparsity on the distance error matrix $\mathbf{S}$. The regularizing term controls how close the perturbed sensors are to their ideal designed locations. Considering that the EDM is invariant with translation, rotation and reflection, we add constraints of references to make sure the solution converges to what we expected in the model. The first constraint requires two neighbor sensors to be close to each other. The second and the third constraints force the center and the orientation of the perturbed sensors are aligned to the ideal designed sensors, respectively. To solve this optimization problem, we use the alternating direction method of multipliers (ADMM), see Appendix 5 for details.

3. NUMERICAL EXPERIMENT

To verify our method, we consider a mono-static radar imaging problem with simulation setup depicted in Fig. 1, in which a total of $N = 51$ sensors are used to image $K = 3$ static objects in the ROI. The mono-static antenna is designed to form a uniform linear array as indicated by the red dots. However, the actual element locations are perturbed, as indicated by the black x-marks, with up to 10 times the center wavelength of the transmitted source signal. The time-domain source pulse is illustrated in Fig. 2 (a), and the received echoes are simulated using the free-space Green's function with added white Gaussian noise, as shown in Fig. 2 (b) .

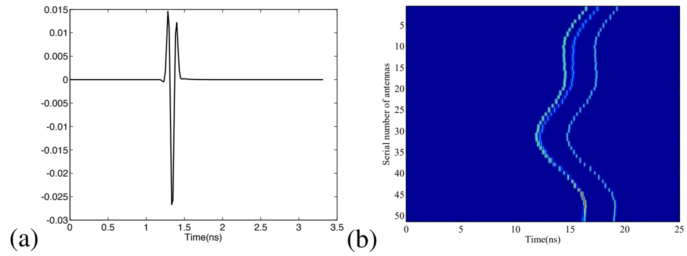

Fig. 2. (a) Source pulse emitted by the transmitter; (b) Simulated radar echoes with noise.

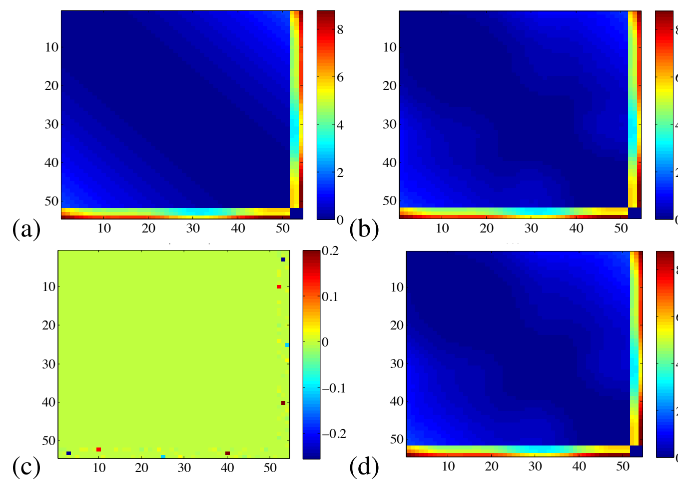

Fig. 3. (a) Estimated EDM using radar echoes and ideal sensor locations, (b) Reconstructed low-rank EDM using our proposed method, (c) Recovered sparse errors of estimated EDM, and (d) True EDM for comparison.

In our sensor localization method, an EDM is first estimated using measured signals and prior information about sensor locations, as shown in Fig. 3 (a). By solving the optimization problem in (12), an underlying low-rank EDM is recovered, as shown in Fig. 3 (b), with sparse error components shown in Fig. 3 (c). For comparison, we present the true EDM of perturbed sensors in Fig. 3 (d). We observe that the recovered EDM is very close to the true EDM, especially those squared distances between sensors. The corresponding recovered sensor and object locations are shown in Fig. 4 (a), which are very well aligned with the true perturbed sensor and object locations. However, if we use the least squares method to estimate sensor locations based on distances observed in $\mathbf{\Phi}$, some of the sensor locations exhibit errors much larger than

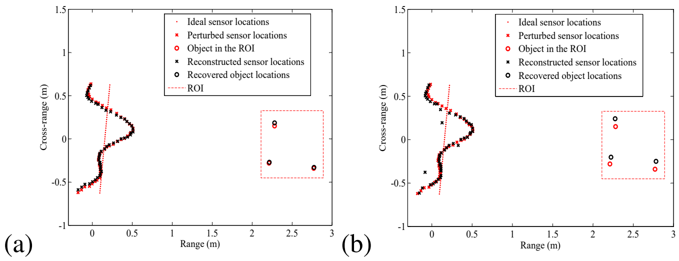

Fig. 4. (a) Estimated locations using our proposed method; (b) Estimated locations using the least squares method.

the source wavelength, as shown in Fig. 4 (b).

4. CONCLUSIONS

We propose a robust sensor localization approach based on Euclidean matrix. Our approach seeks to reconstruct a low-rank Euclidean distance matrix (EDM) of true perturbed sensor locations from a noisy EDM estimate with missing entries using the ADMM method. Simulation results demonstrate that our method significantly improves the performance in localizing sensors of moving platforms compared to the least squares method.

5. APPENDIX

The augmented Lagrangian function is

$$
\begin{aligned}
&\mathcal{L}\left(\mathbf{E}, \mathbf{S}, \{\widetilde{\boldsymbol{r}}_n\}, \{\boldsymbol{l}_k\}, \{\mu^e_{ij}\}, \{\mu^\epsilon_n\}, \mu^c, \mu^o, z_n, \bar{\boldsymbol{r}}, o_{rt}\right) \\
=&\frac{\gamma}{2}\|(\boldsymbol{\Phi} - \mathbf{E} - \mathbf{S}) \odot \mathbf{M}\|_F^2 + |vec\{\mathbf{S} \odot \mathbf{M}\}|_1 \\
&+\frac{\sigma^r}{2}\sum_n \|\widetilde{\boldsymbol{r}}_n - \boldsymbol{r}_n\|^2 + (\|\bar{\boldsymbol{r}} - \frac{1}{N}\sum_n \boldsymbol{r}_n\|^2) + o_{rt}^2 \\
&+\sum_{ij}\left[-\mu^e_{ij}(\mathbf{E}_{ij} - \|\widetilde{\boldsymbol{r}}_i - \widetilde{\boldsymbol{r}}_j\|^2) + \frac{\sigma^e_{ij}}{2}(\mathbf{E}_{ij} - \|\widetilde{\boldsymbol{r}}_i - \widetilde{\boldsymbol{r}}_j\|^2)^2\right] \\
&+\sum_{n=1}^N [-\mu^\epsilon_n(\|\widetilde{\boldsymbol{r}}_n - \widetilde{\boldsymbol{r}}_{n+1}\| + z_n - \epsilon)] \\
&+\sum_{n=1}^N \left[\frac{\sigma^\epsilon_n}{2}(\|\widetilde{\boldsymbol{r}}_n - \widetilde{\boldsymbol{r}}_{n+1}\| + z_n - \epsilon)^2\right] \\
&-\mu^c(\frac{1}{N}\sum_{n=1}^N \|\widetilde{\boldsymbol{r}}_n - \bar{\boldsymbol{r}}\|) + \frac{\sigma^c}{2}(\frac{1}{N}\sum_{n=1}^N \|\widetilde{\boldsymbol{r}}_n - \bar{\boldsymbol{r}}\|)^2 \\
&-\mu^o\left[\sum_{n=1}^N (\widetilde{\boldsymbol{r}}_n - \bar{\widetilde{\boldsymbol{r}}}) \times (\boldsymbol{r}_n - \bar{\boldsymbol{r}}) - o_{rt}\right] \\
&+\frac{\sigma^o}{2}\left[\sum_{n=1}^N (\widetilde{\boldsymbol{r}}_n - \bar{\widetilde{\boldsymbol{r}}}) \times (\boldsymbol{r}_n - \bar{\boldsymbol{r}}) - o_{rt}\right]^2.
\end{aligned}
\tag{14}
$$

The corresponding update equations follow steps

$$
\begin{aligned}
&\text{For } m = 1, ..., M, \\
&\min\mathcal{L}(\cdot) \rightarrow \{\mathbf{E}_m, \mathbf{S}_m, \{\widetilde{\boldsymbol{r}}_n\}_m, \{\boldsymbol{l}_k\}_m, \{\bar{\boldsymbol{r}}\}_m, \{o_{rt}\}_m\} \\
&\{z_n\}_m = \max(\frac{\mu^\epsilon}{\sigma^\epsilon} - \|\widetilde{\boldsymbol{r}}_n - \widetilde{\boldsymbol{r}}_{n+1}\| + \epsilon, 0) \\
&\mu^e_{ij} \leftarrow \mu^e_{ij} - \sigma^e_{ij}(\mathbf{E}_{ij} - \|\widetilde{\boldsymbol{r}}_i - \widetilde{\boldsymbol{r}}_j\|^2) \\
&\mu^\epsilon_n \leftarrow \mu^\epsilon_n - \sigma^\epsilon_n(\|\widetilde{\boldsymbol{r}}_n - \widetilde{\boldsymbol{r}}_{n+1}\| + z_n - \epsilon) \\
&\mu^c \leftarrow \mu^c - \sigma^c(\frac{1}{N}\sum_{n=1}^N \|\widetilde{\boldsymbol{r}}_n - \bar{\boldsymbol{r}}\|) \\
&\mu^o \leftarrow \mu^o - \sigma^o\left(\sum_{n=1}^N (\widetilde{\boldsymbol{r}}_n - \bar{\widetilde{\boldsymbol{r}}}) \times (\boldsymbol{r}_n - \bar{\boldsymbol{r}}) - o_{rt}\right).
\end{aligned}
\tag{15}
$$

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