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Rate-Adaptive LDPC Convolutional Coding with Joint Layered Scheduling and Shortening Design

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Abstract: We propose a joint design method of layered scheduling, shortening and puncturing for LDPC convolutional codes to be scalable across a variety of overhead ranges. Our method achieves greater than 0.4 dB gain over conventional methods.

OCIS codes: (060.4510) Optical communications, (060.1660) Coherent communications, (060.4080) Modulation.

1. Introduction

Modern fiber-optic systems have adopted soft-decision (SD) forward error correction (FEC) based on low-density parity-check (LDPC) codes [1–9]. In the past years, LDPC convolutional codes (LDPC-CC), known as spatially coupled codes, have received a considerable amount of interest because they allow low-latency windowed decoding (WD) [4–9]. For example, Schmalen *et al.* [4–6] achieved excellent performance by designing an irregular degree distribution for LDPC-CC with 1-iteration WD. A further performance improvement was achieved [10] by optimizing layered scheduling [9] for WD in particular for small window sizes.

It is known that optimal degree distribution for irregular LDPC codes depends on the target overhead. It implies that one excellent LDPC code is usually not scalable (compatible) to different overheads. Nevertheless, the next-generation optical networks call for high flexibility with adaptive modulation and coding [11–16]. Rate-adaptive LDPC codes can be realized, e.g., by puncturing [11], shortening [12], and splitting [13]. Puncturing decreases the overhead by omitting parity bit transmission, shortening increases the overhead by omitting transmission of frozen information bits, and splitting modifies parity-check matrix. Particularly, an appropriate selection of shortening [12] (from head of coupling) showed significant gain, yet hardware friendly because nearly zero modification is required for encoding/decoding. In this paper, we further improve the shortening in conjunction with layered scheduling design [10] in more greedy fashion (rather than choosing from only head bits). We verify that our joint design methods achieve significant gain closer to Shannon limit over a variety of FEC overhead ranges.

2. Rate-Adaptive LDPC-CC Design

Bit-interleaved coded-modulation (BICM) using binary SD-FEC and multi-level quadrature-amplitude modulation (QAM) has been used [16] in recent lightwave systems. In Fig. 1, theoretically achievable spectral efficiency (SE) for BICM systems with regular QAMs (from 4-ary to 1024-ary) is present. Depending on channel signal-to-noise ratio (SNR), the best modulation format and FEC overhead that maximizes the SE can be determined as shown in Fig. 2. It is observed that the best FEC overhead significantly varies in particular for lower-order modulations. For example, the optimal FEC overhead ranges from 27% to 189% for 16QAM between 2.5 and 10 dB SNRs. Hence, it is of great importance that practical FEC codes can be seamlessly scalable over a wide range of overhead. Note that small overhead such as 7% is only useful for higher-order QAMs in theory, whereas more practical design of modulation order and FEC overhead is investigated [14, 16].

Fig. 3 depicts quasi-cyclic (QC) LDPC-CCs denoted by a protograph (J,K,L,M), where J is maximum column weight, K is maximum row weight, L is termination length of spatial coupling, and M is the QC lifting size. We use low-latency WD with window size W, for which WM consecutive check nodes (CNs) in total can be activated for every sliding window. Inside the window, we split the CNs into multiple layers for scheduling. Both the size of layers and stride of window sliding are set to be equal to the QC lifting parameter M.

For comparison, we consider conventional round-robin (RR) scheduling, where every layer of M CNs is sequentially updated in a circular manner from the top to the bottom. We optimize the irregular decoding order of the W layers inside the window to improve the decoding convergence speed without requiring any additional complexity. We use protograph-based extrinsic information transfer (P-EXIT) [10] analysis to search for the best proto-CN which can maximize the reliability, in a greedy manner [10]. Note that the reverse RR scheduling [10] was found to be nearly optimal especially for higher overhead cases.

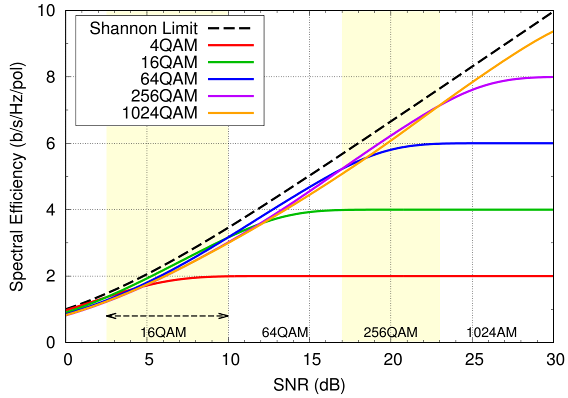


Fig. 1: Achievable SE for BICM systems with QAMs.

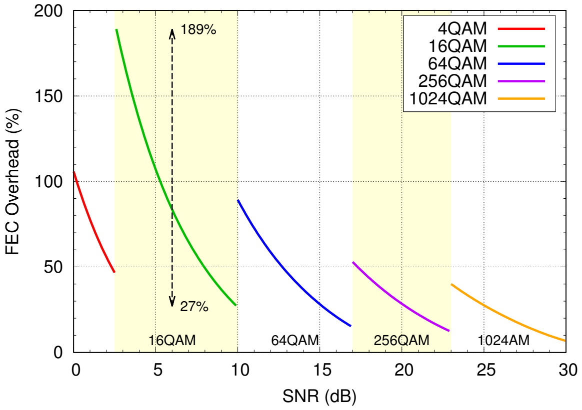


Fig. 2: FEC overhead maximizing SE for QAMs.

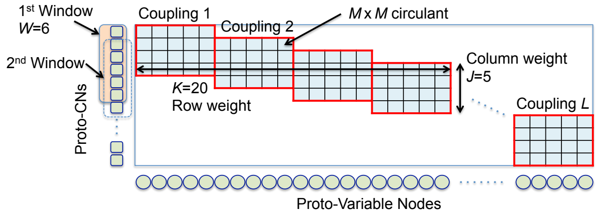


Fig. 3: WD with $W = 6$ for QC-LDPC-CC(J,K,L,M).

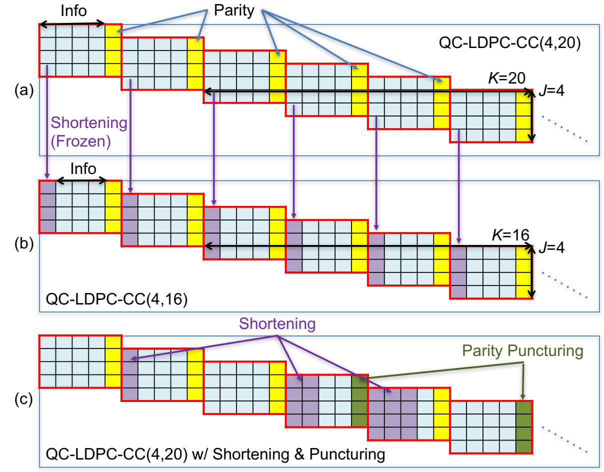


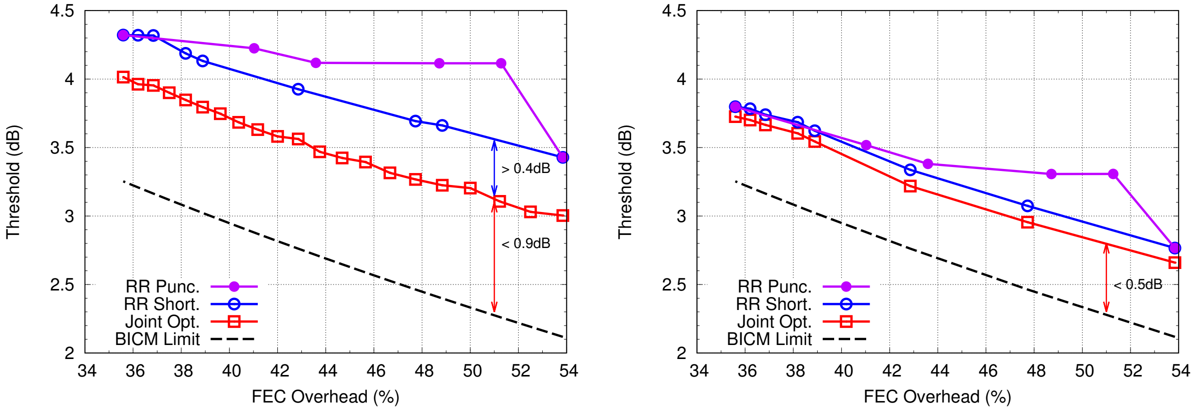
Fig. 4: LDPC-CC with shortening and puncturing: (a) regular LDPC-CC(4,20); (b) regular LDPC-CC(4,16) which is a uniformly shorten version of regular LDPC-CC(4,20); (c) LDPC-CC(4,20) with optimal shortening and puncturing.

In conjunction of layered scheduling design, we optimize the best proto-variable nodes (VNs) to be punctured or shortened to minimize the FEC limit across different overhead. For P-EXIT, channel mutual information are initialized to be 1 (i.e., error free) and 0 (i.e., unreliable) for the proto-VNs shorten and punctured, respectively. Note that LDPC-CC(J,K) can be obtained by a lower-overhead LDPC-CC(J,$K+J$) by uniform shortening as shown in Fig. 4, without need of hardware modification of encoding and decoding. Thus, we can use a mother LDPC-CC having the smallest overhead for joint design of shortening, puncturing, and scheduling. Note that there are a lot of degrees of freedom to choose the proto-VNs to be shorten or punctured, and that uniform shortening such as Fig. 4(b) is not always best. By carefully choosing the proper proto-VNs, wave-like WD message passing can be boosted. In this paper, we optimize the scheduling, shortening, and puncturing for variable-rate LDPC-CC, by means of a greedy tree search [10].

3. Performance Analysis

In order to show the benefit of joint optimization, we first consider the rate adaptation between LDPC-CC(4,16) and LDPC-CC(4,12). Fig. 5(a) shows the achievable threshold as a function of FEC overhead for the number of sub-iterations $I = 2$ in layered scheduling and a window size of $W = 6$. We used 64 survivors when optimizing the layered scheduling. For comparison, we plot RR scheduling with uniformly-distributed puncturing and shortening as well as our joint optimization of scheduling, shortening, and puncturing. Puncturing creates lower overhead from highest-overhead LDPC-CC(4,12), whereas shortening decrease the overhead based on the lowest-overhead LDPC-CC(4,16). It is observed that the proposed method outperforms RR scheduling with shortening by more than 0.4 dB. Even for such a small window size of $W = 6$ and a limited number of iterations of $I = 2$, we can achieve good performance which is within 1 dB of Shannon limit across different overhead.

The gap from Shannon limit can be reduced by increasing iterations or window size. Fig. 5(b) shows the case with $W = 10$. We can see that the threshold for all methods are significantly improved and that the advantage of optimal



(a) $W = 6, I = 2$ (b) $W = 10, I = 2$

Fig. 5: Threshold vs. FEC overhead between truncated LDPC-CC(4,16,20) and LDPC-CC(4,12,20).

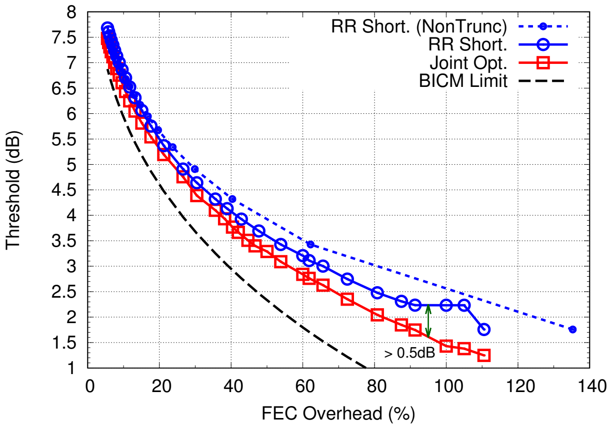


Fig. 6: Threshold achieved by rate-adaptive LDPC-CC for a wide range of overhead ($W = 6, I = 2$).

scheduling over RR becomes marginal. Nonetheless, the proposed method achieves the performance within 0.5 dB of Shannon limit. Note that puncturing does not perform well because mutual information are nullified at punctured proto-VNs in contrast to shortening cases.

The scheduling gain is more significant in higher overhead regimes, where LDPC codes usually do not converge faster [14]. This is confirmed in Fig. 6, showing the remarkable advantage of our joint scheduling and shortening optimization over the conventional RR scheduling with uniform shortening for a wide range of overhead from 5% to 110%. Note that our rate-adaptive LDPC-CC is seamlessly scalable to accommodate the whole overhead without any hardware modification. It is also shown that truncated LDPC-CC significantly outperforms non-truncated ones because of rate loss mitigation.

4. Conclusions

We proposed hardware-friendly rate-compatible LDPC-CC using optimized layered scheduling and shortening. A significant performance improvement of 0.4 dB was achieved for limited decoding iterations and window sizes.

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