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Shulkind, G.; Pajovic, M.; Orlik, P.V.

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Abstract

In various wireless applications, a receiver picks up data packets from multiple users where the packets share a common preamble, but otherwise carry different payloads, are not in temporal sync and are frequency shifted due to Doppler effect and oscillator imperfections. We pose the problem of identifying the number of interfering packets and extracting the payloads as one of finding a sparse representation in a redundant dictionary. However, because of large size of the dictionary due to unknown packet payloads, direct application of conventional recovery methods does not lead to computationally tractable estimation schemes. To overcome this issue, we propose Orthogonal Matching Pursuit with Approximate Atoms (OMP-AA) algorithm aimed to facilitate identification of packet collisions and payload extraction. The simulation study shows that the proposed method performs well compared to an oracle estimator which has perfect knowledge of the packet parameters

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PACKET SEPARATION IN RANDOM ACCESS CHANNELS VIA APPROXIMATE SPARSE RECOVERY

*Gal Shulkind**

Department of EECS
Massachusetts Institute of Technology
Cambridge, MA, 02139, USA
shulkind@mit.edu

Milutin Pajovic, Philip Orlik

Electronics and Communications Group
Mitsubishi Electric Research Laboratories
Cambridge, MA, 02139, USA
{pajovic,porlik}@merl.com

ABSTRACT

In various wireless applications, a receiver picks up data packets from multiple users where the packets share a common preamble, but otherwise carry different payloads, are not in temporal sync and are frequency shifted due to Doppler effect and oscillator imperfections. We pose the problem of identifying the number of interfering packets and extracting the payloads as one of finding a sparse representation in a redundant dictionary. However, because of large size of the dictionary due to unknown packet payloads, direct application of conventional recovery methods does not lead to computationally tractable estimation schemes. To overcome this issue, we propose Orthogonal Matching Pursuit with Approximate Atoms (OMP-AA) algorithm aimed to facilitate identification of packet collisions and payload extraction. The simulation study shows that the proposed method performs well compared to an oracle estimator which has perfect knowledge of the packet parameters.

Index Terms— Source/packet separation, sparse recovery, frequency offset, random access channel, internet of things (IoT)

1. INTRODUCTION

Numerous wireless communication systems involve a receiver picking up data packets from multiple asynchronous users sharing the same channel resource. Many communication protocols for such environments (e.g. IEEE 802.11, IEEE 802.15.4) employ channel access mechanisms, such as Carrier Sense Multiple Access [1] in an attempt to avoid channel collisions and ensure that a single transmission at a time is present on the channel. However, as the density of deployed asynchronous agents increases so does the potential for packet collisions and network congestion, and expected latency may surge. Thus, employing such methods in applications where latency is of critical importance in maintaining safe operation, such as in vehicular communications [2] or for factory automation, may not be appropriate.

Moreover, devices and sensors in the future Internet of Things (IoT) realm of the upcoming 5G standard are expected to share the same channel resource [3]. This is because allocating orthogonal waveforms to a large number of devices within the same cell is impossible since the IoT devices are expected to transmit short messages. Also, only a small fraction of devices in the cell are expected to transmit messages at the same time and thus employing network resource management and control might yield an excessive communication overhead [3]. Therefore, collisions of short packets on a physical layer need to be resolved on the receiver side.

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In lieu of implementing a channel access mechanism, we allow collisions on the channel and propose a receiver-side collision detection and payload extraction scheme capable of recovering some of the collided payload symbols, increasing overall system performance. In an alternative use case, we assume a conventional setup where the channel access mechanism is implemented, but either occasionally fails to prevent packet collisions or grants transmission to a latency-critical packet that happens to collide with a packet already present in the shared channel.

The methods and analysis presented here accommodate scenarios where packets suffer varying time delays and frequency offsets, such as those arising from motion or oscillator variations due to using inexpensive clocks such as in IoT applications. We show that the packet separation problem may be posed as one of finding a sparse representation of the received signal in a redundant dictionary comprised of atoms expressing all possible payload data and channel distortion combinations. In this paper our emphasis is on combating the prohibitive computational complexity of performing packet separation in the sparse representation formalism. In principle, with a dictionary as defined above, the sparse representation problem is amenable to solution by means of generic algorithms such as the Orthogonal Matching Pursuit (OMP) [4] or its variants [5]. However, while theoretically appealing, this approach is computationally intractable as the number of dictionary atoms, and thus the complexity, scale with the number of possible payload configurations per packet, which is exponentially large.

We propose an approach that alleviates the computational load associated with the OMP implementation. In particular, we embed an approximation in the OMP correlation maximization step, where under an assumption of high Signal to Noise and Interference Ratio (SINR) the computational complexity becomes linear in the number of payload symbols per packet. This enables estimation of the packet parameters (delays, frequency offsets and channel gains) and the detection of the payload symbols. The proposed method is tested via simulations and good performance compared to a reference oracle detector that has full knowledge of the packet parameters is achieved.

2. RELATION TO PRIOR WORK

To separate colliding packets in a random access channel, the algorithms in [6, 7] assume the colliding users retransmit packets as many times as the number of collided packets. In addition, the packets employ orthogonal preambles to facilitate collision detection. Also, perfect time synchronization is assumed. In comparison, we aim to separate packets without incurring retransmissions, i.e., from a single received signal realization. In addition, the packets in our framework use the same preamble and are not time synchronized.

A semi-blind method for packet separation using diversity due to multi-path channel, oversampling and/or spreading gain with insertion of known symbols in packets to aid channel estimation has been proposed in [8]. In comparison, we consider packets with the same and contingent preamble and without spreading.

While traditional multi-user detection (MUD) relies on users employing (pseudo-)orthogonal spreading codes [9, 10, 11], a more recent compressive sensing MUD (CS-MUD) research considers scenarios where only a small portion, out of a large number of users, simultaneously transmit packets and exploits sparse recovery methods to detect active users and transmitted messages. As such, [12, 13, 14] assume perfect synchronization among users, while [15] considers asynchronous users and estimates their delays. In comparison to these works, where the users are also equipped with unique signature sequences or the packets are precoded before transmission [16], each packet in our setup has the same preamble and unknown data content, not subject to a spreading/precoding.

In a separate line of research, a vast literature considers a sinusoids separation problem, starting from the MUSIC algorithm [17], until recent off-the-grid optimization-based approaches exploiting frequency domain sparsity [18, 19] and a practical OMP-like recovery with frequency estimate refinement step [20]. Although we formulate the packet separation as a problem of sparse recovery in the delay and frequency offset domain, we emphasize that the considered problem is more challenging in a sense that each sinusoid carries unknown payload symbols and experiences an unknown delay.

3. PROBLEM FORMULATION

A wireless receiver picks up S data packets arriving from separate asynchronous transmitters. The kth packet is composed of a fixed preamble $p(t)$, identical for all packets, followed by a packet specific payload $g_k(t)$. The preamble consists of M_1 pre-specified and known symbols $\{p_m\}$ while the payload consists of M_2 additional data symbols $\{g_{km}\}$ all chosen from the same constellation $p_m, g_{km} \in \mathcal{G}$. Denoting the pulse shaping function with $h(t)$ and the symbol time with T_c, we have

$$\begin{aligned} p(t) &\equiv \sum\nolimits_{m=0}^{M_1-1} p_m h(t-mT_c) \\ g_k(t) &\equiv \sum\nolimits_{m=0}^{M_2-1} g_{km} h(t-(m+M_1)T_c) \end{aligned} \quad (1)$$

Due to channel conditions each received packet experiences different time delay τ_k, frequency offset ω_k and complex fading gain a_k, which are unknown to the receiver. The received signal is a superposition of the collided packets and is additionally corrupted by additive noise $n(t)$. Without loss of generality we assume the receiver records measurements at times $t=0, 1, \ldots, T$ such that the received signal $s(t)$ is

$$s(t) = \sum_{k=1}^{S} a_k \left[p(t-\tau_k) + g_k(t-\tau_k)\right] e^{j\omega_k t} + n(t) \quad (2)$$

To simplify the exposition, we take the pulse shaping function to be a rectangular window of length T_C, $h(t) = \mathbb{1}_{[0,T_c]}(t)$ and also assume binary phase shift keying (BPSK) modulation format, $\mathcal{G}=\{-1, +1\}$. Our method can be applied to other symbol sets and pulse shapes as well. In fact, we validate the proposed algorithm using the GMSK modulation. The noise is circularly symmetric complex white Gaussian (AWGN) of zero mean and variance σ^2.

In this paper, our goal is to estimate the number of received data packets S, their parameters $\{a_k, \tau_k, \omega_k\}$, as well as the payload symbols $\{g_{km}\}$ from the received signal $s(t)$. Since the Maximum Likelihood (ML) approach for this problem is computationally intractable we focus on developing approximate techniques to facilitate tractable parameter extraction.

4. EXTRACTION ALGORITHM

In this section we formulate collision detection (determining S) and payload extraction (retreiving $\{g_{km}\}$) associated with model (2) as a problem of sparse representation in a redundant dictionary. We define a dictionary $\mathcal{D} \equiv \{d_{\tau,\omega,\mathbf{g}}(t)\}$ whose atoms include all possible individual received packets, parametrized by the channel parameters (τ, ω) and the packet payload $\mathbf{g} \equiv [g_0, \cdots, g_{M_2-1}]$. Specifically, atom $d_{\tau,\omega,\mathbf{g}}(t)$ corresponding to a packet with delay τ, frequency offset ω and payload $\mathbf{g}$ is defined according to

$$d_{\tau,\omega,\mathbf{g}}(t) \equiv \left[p(t-\tau) + \sum_{m=0}^{M_2-1} g_m h(t-\tau-(m+M_1)T_c)\right] e^{j\omega t} \quad (3)$$

Using (3), the receiver signal in (2) can be expressed as a linear combination of exactly S atoms (packets)

$$s(t) = \sum_{k=1}^{S} a_k d_{\tau_k,\omega_k,\mathbf{g}_k}(t) + n(t) \quad (4)$$

Note that τ is determined to within the sampling period resolution such that it belongs to a finite set of size $O(T)$ whereas $\mathbf{g}$ belongs to a finite set of size $O(2^{M_2})$ (all possible binary payload sequences) and the frequency offsets ω belong to a continuous range of possible offsets, assumed to be specified in advance in compliance with some physical constraints, and discretized on a grid of size W. Thus, the number of dictionary atoms in $\mathcal{D}$ scales as $O(WT2^{M_2})$, i.e. exponential in the number of payload symbols per packet M_2.

Eq. (4) suggests that for small S the payload extraction problem can be posed as a sparse representation and one may employ, e.g. the OMP algorithm [4] to recover the atoms. However, as the computational complexity of the OMP scales with the dictionary size, it becomes prohibitive as M_2 increases due to the aforementioned exponential dependency.

To see this concretely consider the standard implementation of the OMP algorithm. The main computational step is calculation of correlations between a residual signal $r(t)$ and all dictionary atoms, at each iteration finding the atom satisfying $\tau^\star, \omega^\star, \mathbf{g}^\star = \operatorname{argmax}_{\tau,\omega,\mathbf{g}} |\langle r(t), d_{\tau,\omega,\mathbf{g}}(t)\rangle|^2$. Substituting (3)

$$\tau^\star, \omega^\star, \mathbf{g}^\star = \operatorname*{argmax}_{\tau,\omega,\mathbf{g}} |r^{(p)}(\tau,\omega) + \sum_{m=0}^{M_2-1} \bar{g}_m r_m^{(h)}(\tau,\omega)|^2 \quad (5)$$

where $\bar{(\cdot)}$ denotes complex conjugation and we have defined

$$\begin{aligned} r^{(p)}(\tau,\omega) &\equiv \langle r(t), p(t-\tau)e^{j\omega t}\rangle \\ r_m^{(h)}(\tau,\omega) &\equiv \langle r(t), h(t-\tau-(m+M_1)T_c)e^{j\omega t}\rangle \end{aligned} \quad (6)$$

For fixed (τ, ω), maximizing over $\mathbf{g}$ in (5) is NP-hard in the general case as this can be converted to the standard form of a binary quadratic program with arbitrary complex coefficients [21]. Thus, this step becomes computationally intractable as the number of payload bits M_2 increases.

In the sequel we propose an approximate relaxation to OMP, circumventing the prohibitive optimization step (5).

4.1. OMP with Approximate Atoms

The exponential dependency of $|\mathcal{D}|$ in M_2 renders OMP computationally intractable for our problem. In this section we devise an approximation algorithm that draws on conventional OMP, however instead of working with the fixed large dictionary $\mathcal{D}$ we iteratively and efficiently build on the fly an approximated dictionary $\tilde{\mathcal{D}}$ of manageable size.

Our strategy will be as follows. We start with an empty set $\tilde{\mathcal{D}} = \emptyset$. At each OMP iteration step, instead of correlating the residual $r(t)$ with all possible atoms $d_{\tau,\omega,\mathbf{g}}(t) \in \mathcal{D}$, we split the maximization step (5). First, for each (τ, ω) we efficiently find an approximate solution for the optimal sequence $\mathbf{g}^\star(\tau, \omega)$ and then perform the maximization over (τ, ω) to find the approximately best correlated dictionary element $\tilde{d}_{\tau^\star,\omega^\star,\mathbf{g}^\star(\tau^\star,\omega^\star)}(t)$ which is added to $\tilde{\mathcal{D}}$. The algorithm follows as usual with the received signal $s(t)$ represented via an optimal linear combination of atoms from $\tilde{\mathcal{D}}$ in least squares sense. The representation error is the new residual signal, which is fed back to the next iteration.

We will shortly see that the derived approximations will result in linear computational complexity in M_2. The proposed method, which we refer to as the OMP with Approximated Atoms (OMP-AA), is justified next and summarized in Algorithm 1.

To see how the OMP-AA plays out, consider an idealized setting where at the beginning of an OMP-AA iteration there is just one dominant atom, parametrized with $(\hat{\tau}, \hat{\omega}, \hat{a}, \hat{\boldsymbol{g}})$, present in the residual signal $r(t)$

$$r(t) = \hat{a}\left[p(t-\hat{\tau}) + \sum_{m=0}^{M_2-1} \hat{g}_m h(t-\hat{\tau}-(m+M_1)T_c)\right] e^{j\hat{\omega}t} \quad (7)$$

substituting this in (6) we have at $(\tau, \omega) = (\hat{\tau}, \hat{\omega})$

$$r^{(p)}(\hat{\tau},\hat{\omega}) = \hat{a}\|p\|_2^2, \qquad r_m^{(h)}(\hat{\tau},\hat{\omega}) = \hat{a}\hat{g}_m\|h\|_2^2 \quad (8)$$

such that

$$\underset{g_m}{\operatorname{argmax}} |r^{(p)}(\hat{\tau},\hat{\omega}) + \sum_{m=0}^{M_2-1} \bar{g}_m r_m^{(h)}(\hat{\tau},\hat{\omega})|^2 = \frac{r_m^{(h)}(\hat{\tau},\hat{\omega})}{r^{(p)}(\hat{\tau},\hat{\omega})} \frac{\|p\|_2^2}{\|h\|_2^2} \quad (9)$$

For the OMP-AA algorithm we will extend this result to have

$$g_m^\star(\tau,\omega) = \mathbb{P}_{\mathcal{G}}\left(\frac{r_m^{(h)}(\tau,\omega)}{r^{(p)}(\tau,\omega)} \frac{\|p\|_2^2}{\|h\|_2^2}\right) \quad (10)$$

where $\mathbb{P}_{\mathcal{G}}(\cdot)$ is a projection on the symbol set $\mathcal{G}$.

This approximation is exact for $(\tau,\omega) = (\hat{\tau},\hat{\omega})$ and under high SINR conditions, where $r(t)$ comprises a single dominant packet with parameters $(\hat{\tau},\hat{\omega})$. For (τ,ω) shifted with respect to those of the dominant residual packet $(\hat{\tau},\hat{\omega})$, (10) becomes less accurate, but assuming the packet at $(\hat{\tau},\hat{\omega})$ is dominant the OMP-AA optimizing over (τ,ω) will result in tuning on the correct parameters and using $g_m^\star(\hat{\tau},\hat{\omega})$ making these inaccuracies inconsequential.

Finally for the second step of OMP-AA, performing the outer optimization over (τ,ω) following the previous approximation we made for $g_m^\star(\tau,\omega)$ reads

$$\tau^\star, \omega^\star = \underset{\tau,\omega}{\operatorname{argmax}} |r^{(p)}(\tau,\omega) + \sum_m g_m^\star(\tau,\omega) r_m^{(h)}(\tau,\omega)|^2 \quad (11)$$

Algorithm 1 OMP with Approximated Atoms

function OMP-AA(s, ϵ)
  $r = s$, and $\tilde{\mathcal{D}} = \{\emptyset\}$
  while $\|r\|_2^2 > \epsilon\|s\|_2^2$ **do**
    for all (ω,τ) pairs **do**
      Evaluate $r_m^{(h)}(\tau,\omega)$ and $r^{(p)}(\tau,\omega)$ (Eq. (6))
      Evaluate $\forall m : g_m^\star(\tau,\omega)$ (Eq. (10))
    $\tau^\star, \omega^\star = \underset{\tau,\omega}{\operatorname{argmax}} |r^{(p)} + \sum_m g_m^\star r_m^{(h)}|^2$ (Eq. (11))
    Set of recovered atoms: $\tilde{\mathcal{D}} = \tilde{\mathcal{D}} \cup \tilde{d}_{\tau^\star,\omega^\star,\mathbf{g}^\star(\tau^\star,\omega^\star)}$
    Least squares estimate of weights: $a^\star = \tilde{\mathcal{D}}^+ s$
    Evaluate residual: $r = s - \tilde{\mathcal{D}} a^\star$
  return $\tilde{\mathcal{D}}, a^\star$

We solve (11) brute force, attempting all (τ,ω) value combinations in succession. Notice that our combined procedure (step (11) followed by (10)) has linear complexity in M_2, W and T, thus alleviating the previous exponential dependency as we intended.

The OMP-AA scheme relies on the high SINR approximation which holds when one atom at a time dominates the residual $r(t)$. The accuracy of the OMP-AA can be augmented by treating the estimates for S, $\{a_k^\star, \tau_k^\star, \omega_k^\star\}$ fixed and known and performing the inference just over the payload symbols $\{g_m^\star\}$ in the second stage. This can be done by means of a graphical model with a hidden layer representing symbol values across packets and an observed layer representing measurements $s(t)$ and running the max-product algorithm [22]. The empirical evaluation validates this approach, but the details are skipped due to space constraints.

4.2. Performance Guarantees and Number of Packets Detection

Regarding theoretical performance guarantees pertaining to sparse recovery algorithms, the main body of research has focused on noncoherent dictionaries, e.g., in [4]. In [23], the most relevant to our setting, the authors study sparse approximation algorithms under an assumption of limited pairwise coherence between participating atoms. They show that by adapting the OMP algorithm to exclude coherent atoms from participating in the reconstruction, accurate detection to within a prescribed coherence region around the underlying atoms can be guaranteed if certain conditions on the noise power, dynamic range of the signal and sparsity levels are met (see Thm. 1 in [23]). While the predictions of [23] are pessimistic as they guarantee performance for all noise realizations they hint at the role high signal to noise ratio, small dynamic range and small pairwise coherence between underlying atoms play in enabling greedy atom identification and reliable reconstruction.

Ported to our problem setting, the results of [23] would guarantee accurate detection of atoms under favorable conditions where channel parameter for different data packets are sufficiently different ensuring low pairwise coherence. However, as the underlying sparsity level is generally not known in our setting we resort to heuristics in order to sort the reconstructed OMP atoms into either unique data packets or noise-describing atoms. Specifically, we assume that the maximum power ratio between packets in the underlying signal does not exceed $r_{\max}$, and additionally that the maximum pairwise coherence between atoms is $\eta_{\max}$. After the OMP-AA algorithm stops, we measure the power of each reconstructed atom as well as its maximal pairwise correlation with previously detected atoms. We tag only those atoms whose power levels are stronger than a fraction $r_{\max}$ of the most powerful atom detected so far, and whose correla-

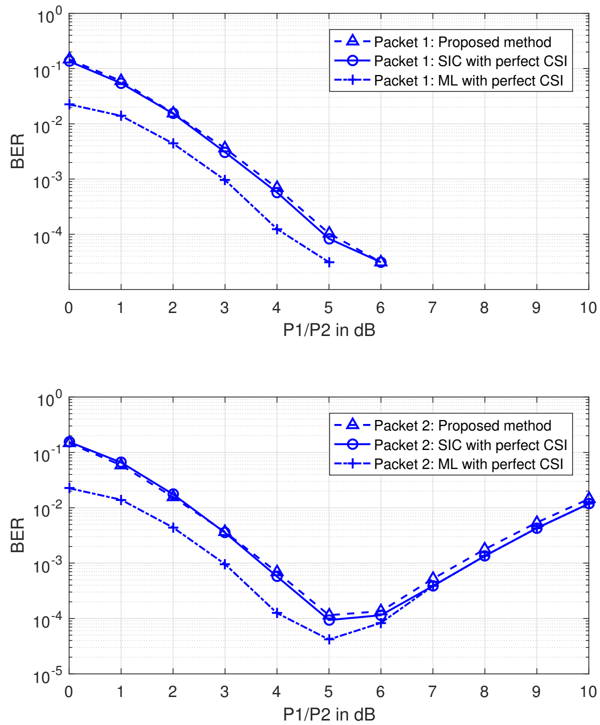

Fig. 1: BER performance of the proposed algorithm and SIC and ML with perfect knowledge of packet parameters for $f_2 = 0.1$.

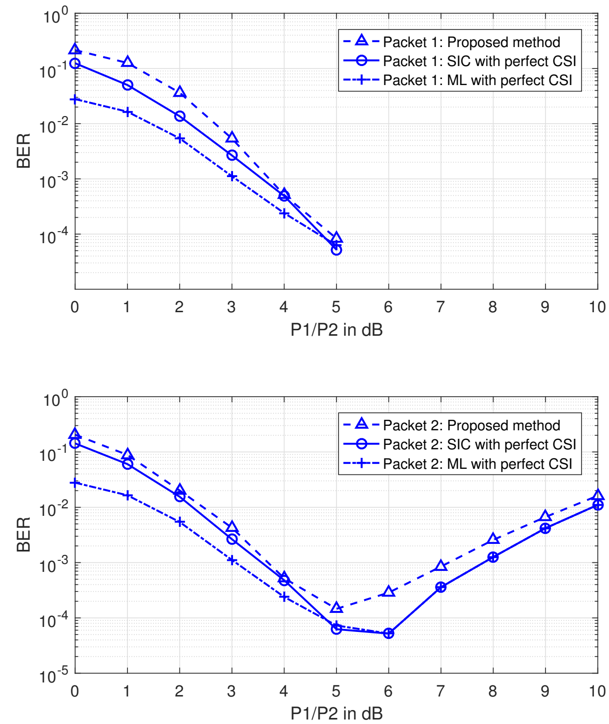

Fig. 2: BER performance of the proposed algorithm and SIC and ML with perfect knowledge of packet parameters for $f_2 = 0.0125$.

tion with previously detected atoms are weaker than $\eta_{\max}$ as atoms corresponding to data packets.

5. SIMULATION RESULTS

The OMP-AA algorithm is validated using Monte-Carlo simulations. We consider a two packet arrival scenario with packet delays τ_1=τ_2=7, meaning that they completely overlap in time. The normalized frequency offset, defined as $f = \omega T_c/2\pi$, of Packet 1 is $f_1 = 6.4 \times 10^{-4}$, chosen to be off-the-grid (251 points in the frequency offset domain between -0.1 and 0.1). Two different values for the normalized frequency offset of Packet 2 are considered such that the two packets are in ω domain relatively separated in one case, and relatively close in the other case. The received power of Packet 1 is fixed to 0 dB, while the received power of Packet 2, P_2, is subject to a simulation sweep. We assume the received power of each packet also incorporates the corresponding channel gain, so that the simulated channels have unit magnitudes and random phases (uniformly distributed in $[-\pi, \pi)$). The variance of the AWGN is -15 dB. Each packet consists of known 32-bit preamble and random 192-bit payload. We use the IoT-friendly Gaussian Minimum Shift Keying (GMSK) modulation [24] with memory, with bandwidth-time product $BT = 0.5$ and oversampling rate of 4.

The bit error rate (BER) corresponding to payload bits of the packets recovered in the first two iterations of the OMP-AA is used as a performance metric. As benchmarks, we also simulate the performance of the Maximum Likelihood (ML) (i.e., the Viterbi algorithm), and Successive Interference Cancellation (SIC) detection with perfect channel state information (CSI). In other words, the ML and SIC receivers are supplied with the true values of packet delays, frequency offsets and corresponding channel coefficients.

The comparisons of the measured payload detection BER's are shown in Figures 1 and 2, for normalized frequency offsets $f_2 = 0.1$ and $f_2 = 0.0125$, respectively. The number of Monte-Carlo runs used to generate the plots is 1000, meaning that the minimum measurable BER (obtained with a single bit error) is 5.2×10^{-6}. All payload bits of Packet 1 are correctly detected over 1000 runs when P_1/P_2 is greater than 6 dB for $f_2 = 0.1$, and greater than 5 dB for $f_2 = 0.0125$.

As can be seen, the proposed algorithm has insignificant performance degradation with respect to the SIC with perfect CSI when $f_2 = 0.1$, indicating that its parameter estimates are fairly accurate. As can be expected, the performance loss with respect to the SIC with perfect CSI increases when the two packets get closer in the frequency offset domain. However, the BER performance of the proposed algorithm is still relatively close to the measured BER corresponding to the SIC with perfect CSI even for $f_2 = 0.0125$, validating the packet separation capability of the proposed algorithm.

6. CONCLUSION

We have proposed OMP-AA algorithm for estimating delays, frequency offsets, channel coefficients and payload symbols of collided packets in a random access channel. The algorithm builds upon the conventional OMP and introduces a payload-centric iterative dictionary approximation. The simulations show an insignificant performance degradation of the OMP-AA with respect to an oracle with perfect knowledge of all parameters of the colliding packets.

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