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TR2016-044 July 2016

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*2016 American Control Conference (ACC)*

# Further Results and Properties of Indirect Adaptive Model Predictive Control for Linear Systems with Polytopic Uncertainty

Donglei Fan, Stefano Di Cairano

***Abstract*— We extend a recently developed design for indirect adaptive model predictive control (IAMPC) and presents additional results on its stability properties. IAMPC guarantees constraints satisfaction including during the learning transient, is input-to-state stable (ISS) with respect to the parameter estimation error, and has computational burden comparable to that of non-adaptive MPC. In this paper we extend IAMPC to the case of uncertain input-to-state matrix, we provide a new method to design robust constraints, and we show additional stability results, in particular that asymptotic stability does not require the parameter estimation error to be zero, which also allow us to derive a tighter ISS Lyapunov function.**

## I. INTRODUCTION

The interest on model predictive control (MPC) in several applications domains [1]–[3] is due to its capability of achieving high performance control for multivariable systems subject to constraints. However, MPC requires a reliable prediction model, which may be hard to obtain before controller deployment, especially in applications domains such automotive, factory automation, and aerospace [2], [3], due to part-to-part variability, aging, and manufacturing imprecisions. Thus, often MPC needs to operate with uncertain models and, for cost and verification requirements, it needs to be restricted to execute with limited computational effort.

When the model parameters are unknown but constant or slowly varying, a robust MPC approach [4]–[6] may be unnecessarily conservative and computationally expensive, e.g., due to assuming continuous changes in the parameters or requiring solving linear matrix inequalities (LMIs). Instead the uncertain parameters can be learned and the prediction model, constraints and cost function corrected accordingly, resulting in adaptive MPC. Some adaptive MPC algorithms have been recently proposed [7]–[9], based on different model assumptions and computational frameworks.

In [10], an Indirect Adaptive MPC (IAMPC) method was proposed for uncertain systems modeled as polytopic linear difference inclusions (pLDIs), where the uncertainty is associated to the convex combination vector by which the vertex models of the pLDI are combined to produce the actual system dynamics. The unknown vector is assumed to be constant or slowly varying, motivating the use of adaptation. To achieve a computational burden similar to standard MPC, IAMPC only solves online a quadratic programming (QP) problem, exploits robust control invariant (RCI) sets [11] to enforce constraint satisfaction, and terminal cost and set designed from parameter-dependent Lyapunov functions [12], [13] for obtaining stability properties. The plant in closed-loop with IAMPC satisfies input and state constraint even during the learning transient, and is input-to-state stable (ISS) with respect to the parameter estimation error. Since ISS "parametrizes" the closed-loop behavior with respect to the estimation error, IAMPC makes only minimal assumptions on the estimator, basically, that the estimator provides a convex combination vector, which allows to separate the estimator and the control design.

In this paper, we extend IAMPC to the uncertainty present also in the input-to-state matrix (i.e., $B$), and show that with appropriate changes to the design procedures, the IAMPC properties still hold. Then, we prove a more stringent stability result, where asymptotic stability (AS) is achieved also with a small, yet, non-zero, estimation error, which results in a tighter ISS Lyapunov function. Also, we propose an alternative method for designing robust constraints to the maximal RCI set in [10], which results in a computationally simpler, albeit slightly more conservative, design.

In the rest of the paper, in Section II we describe the IAMPC and the related control problem. In Section III we extend the unconstrained IAMPC design to the case of uncertainty in the input-to-state matrix, and derive tighter stability results showing AS even under a (small) non-zero estimation error. In Section IV we describe the new design for the robust constraint for constrained IAMPC. In Section V we show a numerical example and a case study in satellite orbit control. Conclusion are drawn in Section VI.

*Notation:* $\mathbb{R}$, $\mathbb{R}_{0+}$, $\mathbb{R}_{+}$, $\mathbb{Z}$, $\mathbb{Z}_{0+}$, $\mathbb{Z}_{+}$ are the sets of real, nonnegative real, positive real, and integer, nonnegative integer, positive integer numbers. We denote intervals using notations like $\mathbb{Z}_{[a,b)} = \{z \in \mathbb{Z} : a \leq z < b\}$. $\mathrm{co}\{\mathcal{X}\}$ denotes the convex hull of the set $\mathcal{X}$, and $\mathrm{int}(\mathcal{X})$ its interior. For vectors, inequalities are intended componentwise, while for matrices indicate (semi)definiteness, and $\lambda_{\min}(Q)$ denotes the smallest eigenvalue of $Q$. By $[x]_i$ we denote the $i$-th component of vector $x$, and by $I$ and $0$ the identity and the "all-zero" matrices of appropriate dimension. $\|\cdot\|_p$ denotes the $p$-norm, and $\|\cdot\| = \|\cdot\|_2$. $B_r(0) \subset \mathbb{R}^n$ denotes the open ball centered at the origin with radius $r$. For a discrete-time signal $x \in \mathbb{R}^n$ with sampling period $T_s$, $x(t)$ is the state a sampling instant $t$, i.e., at time $T_s t$, $x_{k|t}$ denotes the predicted value of $x$ at sample $t + k$, i.e., $x(t + k)$, based on data at sample $t$, and $x_{0|t} = x(t)$. A function $\alpha : \mathbb{R}_{0+} \to \mathbb{R}_{0+}$ is of class $\mathcal{K}$ if it is continuous, strictly increasing, $\alpha(0) = 0$; if in addition $\lim_{c\to\infty} \alpha(c) = \infty$, $\alpha$ is of class $\mathcal{K}_\infty$.

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## II. Preliminaries and Problem Definition

We introduce definitions and results used in the subsequent developments, see, e.g., [14, Appendix B] for details.

*Definition 1:* Given $x(t+1) = f(x(t), w(t))$, $x \in \mathbb{R}^n$, $w \in \mathcal{W} \subseteq \mathbb{R}^d$, a set $\mathcal{S} \subset \mathbb{R}^n$ is robust positive invariant (RPI) for $f$ iff for all $x \in \mathcal{S}$, $f(x,w) \in \mathcal{S}$, for all $w \in \mathcal{W}$. If $w = \{0\}$, $\mathcal{S}$ is called positive invariant (PI). $\square$

*Definition 2:* Given $x(t+1) = f(x(t), u(t), w(t))$, $x \in \mathbb{R}^n$, $u \in \mathcal{U} \subseteq \mathbb{R}^m$, $w \in \mathcal{W} \subseteq \mathbb{R}^d$, a set $\mathcal{S} \subset \mathbb{R}^n$ is robust control invariant (RCI) for $f$ iff for all $x \in \mathcal{S}$, there exists $u \in \mathcal{U}$ such that $f(x,u,w) \in \mathcal{S}$, for all $w \in \mathcal{W}$. If $w = \{0\}$, $\mathcal{S}$ is called control invariant (CI). $\square$

*Definition 3:* Given $x(t+1) = f(x(t))$, $x \in \mathbb{R}^n$, and a PI set $\mathcal{S}$ for $f$, with $0 \in \mathcal{S}$, a function $\mathcal{V} : \mathbb{R}^n \to \mathbb{R}_{0+}$ such that there exists $\alpha_1, \alpha_2, \alpha_\Delta \in \mathcal{K}_\infty$ and $\alpha_1(\|x\|) \leq \mathcal{V}(x) \leq \alpha_2(\|x\|)$, $\mathcal{V}(f(x)) - \mathcal{V}(x) \leq -\alpha_\Delta(\|x\|)$ for all $x \in \mathcal{S}$ is a Lypaunov function for $f$ in $\mathcal{S}$. $\square$

*Definition 4:* Given $x(t+1) = f(x(t), w(t))$, $x \in \mathbb{R}^n$, $w \in \mathcal{W} \subseteq \mathbb{R}^d$, and a RPI set $\mathcal{S}$ for $f$, with $0 \in \mathcal{S}$, a function $\mathcal{V} : \mathbb{R}^n \to \mathbb{R}_+$ such that there exists $\alpha_1, \alpha_2, \alpha_\Delta \in \mathcal{K}_\infty$ and $\gamma \in \mathcal{K}$ such that $\alpha_1(\|x\|) \leq \mathcal{V}(x) \leq \alpha_2(\|x\|)$, $\mathcal{V}(f(x)) - \mathcal{V}(x) \leq -\alpha_\Delta(\|x\|) + \gamma(\|w\|)$ for all $w \in \mathcal{W}$ and for all $x \in \mathcal{S}$ is an input-to-state stable (ISS) Lyapunov function for $f$ in $\mathcal{S}$ with respect to $w$. $\square$

*Result 1:* Given $x(t+1) = f(x(t))$, $x \in \mathbb{R}^n$, and a PI $\mathcal{S}$ for $f$, with $0 \in \mathcal{S}$, if there exists a Lyapunov function for $f$ in $\mathcal{S}$, the origin is asymptotically stable (AS) for $f$ with domain of attraction $\mathcal{S}$. Given $x(t+1) = f(x(t), w(t))$, $x \in \mathbb{R}^n$, $w \in \mathcal{W} \subseteq \mathbb{R}^d$, and a RPI $\mathcal{S}$ for $f$, with $0 \in \mathcal{S}$, if there exists a ISS Lyapunov function for $f$ in $\mathcal{S}$, the origin is ISS for $f$ with respect to $w$ with domain of attraction $\mathcal{S}$. $\square$

### A. Indirect adaptive model predictive control

IAMPC was developed in [10] for systems with uncertainty in the state matrix. Here we extend the framework to the case where the uncertainty is present also in the input matrix. Thus, we consider the class of constrained uncertain discrete-time systems

$$x(t+1) = \sum_{i=1}^{\ell} [\bar{\xi}]_i (A_i x(t) + B_i u(t)), \tag{1a}$$
$$x \in \mathcal{X},\ u \in \mathcal{U} \tag{1b}$$

where $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $i \in \mathbb{Z}_{[1,\ell]}$ are known matrices, and $\mathcal{X} \subseteq \mathbb{R}^n$, $\mathcal{U} \subseteq \mathbb{R}^m$ are constraints on system states and inputs. In (1), the uncertainty is associated to $\bar{\xi} \in \Xi \subset \mathbb{R}^\ell$, which is unknown and constant or changing much more slowly than the system dynamics, and $\Xi = \{\xi \in \mathbb{R}^\ell : 0 \leq \xi \leq 1, \sum_{i=1}^{\ell} [\xi]_i = 1\}$. Essentially, $\Xi$ contains all convex combination vectors of dimension $\ell$ for the vertex systems $A_i x(t) + B_i u(t)$.

It is assumed that an estimator generates a (time varying) estimate $\xi(t)$ of $\bar{\xi}$ such that $\xi(t) \in \Xi$ for all $t \in \mathbb{Z}_{0+}$, and we define the (parameter) estimation error

$$\tilde{\xi}_t = \bar{\xi} - \xi_t,\ \tilde{\xi}(t) \in \tilde{\Xi}(\xi_t) \tag{2}$$

where $\tilde{\Xi}(\xi) = \{\tilde{\xi} :\ \bar{\xi} - \xi, \bar{\xi} \in \Xi\}$ is the set of admissible estimation errors. The IAMPC does not require a specific estimator choice, but only that $\xi(t) \in \Xi$ for all $t \in \mathbb{Z}_{0+}$.

At time $t \in \mathbb{Z}_{0+}$, given the sequence of estimates $\{\xi(t)\}_t$, the IAMPC constructs estimate prediction sequence $\xi_t^N \in \Xi^{N+1}$ and solves the finite time optimal control problem

$$\mathcal{V}^{\mathrm{MPC}}_{\xi_t^N}(x(t)) = \tag{3a}$$
$$\min_{U_t} \quad x'_{N|t}\mathcal{P}(\xi_{N|t})x_{N|t} + \tag{3b}$$
$$\sum_{k=0}^{N-1} x'_{k|t}Qx_{k|t} + u'_{k|t}Ru_{k|t} \tag{3c}$$
$$\text{s.t.} \quad x_{k+1|t} = \sum_{i=1}^{\ell} [\xi_{k|t}]_i (A_i x_{k|t} + B_i u_{k|t}) \tag{3d}$$
$$(x_{k|t}, u_{k|t}) \in \mathcal{C}_{xu}, k \in \mathbb{Z}_{[0,N-1]} \tag{3e}$$
$$x_{N|t} \in \mathcal{X}_N \tag{3f}$$
$$x_{0|t} = x(t), \tag{3g}$$

where $N \in \mathbb{Z}_+$ is the prediction horizon, $Q \in \mathbb{R}^{n\times n}$, $R \in \mathbb{R}^{m\times m}$, $Q, R > 0$, $\mathcal{P}(\xi) \in \mathbb{R}^{n\times n}$, $\mathcal{P}(\xi) > 0$, for all $\xi \in \Xi$, $\mathcal{C}_{xu} \subseteq \mathcal{X} \times \mathcal{U}$, $U_t = [u_{0|t}\ \ldots u_{N-1|t}]$ is the sequence of control inputs along the prediction horizon, and $U_t^* = [u^*_{0|t}\ \ldots u^*_{N-1|t}]$ is the optimal solution of (3). The control input at time $t \in \mathbb{Z}_{0+}$ by IAMPC is then $u(t) = u^*_{0|t}$. The IAMPC design problem can be formalized as follows.

*Problem 1:* Given (1) and an estimator producing $\{\xi(t)\}_t$ such that $\xi(t) \in \Xi$ for all $t \in \mathbb{Z}_{0+}$, design the (causal) sequence of predicted convex combination vectors $\xi_t^N$, the terminal cost $\mathcal{P}(\xi)$, the robust terminal set $\mathcal{X}_N$, and the robust constraint set $\mathcal{C}_{xu}$ in (3) so that the IAMPC that at any $t \in \mathbb{Z}_{0+}$ solves (3) and applies $u(t) = u^*_{0|t}$ achieves: $(i)$ ISS of the closed-loop with respect to $\tilde{\xi}_{0|t}$, $(ii)$ robust satisfaction of the constraints including when $\tilde{\xi}_{0|t} \neq 0$, $(iii)$ runtime computational load comparable to a (non-adaptive) MPC requiring only the solution of QPs, $(iv)$ AS of the origin of the closed-loop system when $\|\xi(t) - \bar{\xi}\|_1 \leq \Delta$ for some $\Delta > 0$. $\square$

In [10] a design that achieves $(i)-(iii)$ of Problem 1 was proposed for when $B_i = B$ for all $i \in \mathbb{Z}_{[1,\ell]}$ in (1). Next we modify such design to overcome such restriction, which results in a more conservative design for the terminal set, but enables enable an alternative simpler calculation of the robust constraints $\mathcal{C}_{xu}$. We show here that the design achieves also $(iv)$. As in [10] we select as parameter prediction rule

$$\xi_{k|t} = \xi(t - N + k),\ \forall k \in \mathbb{Z}_{[0,N]}, \tag{4}$$

ensuring $\xi_{k|t} = \xi_{k+1|t-1}$, for all $t \in \mathbb{Z}_+$, $k \in \mathbb{Z}_{[0,N-1]}$.

In what follows only sketches of the proofs of the main results are shown, due to limited space.

## III. Unconstrained IAMPC: Design and Stability Results

Due to the increased uncertainty in (1), the procedure for the design of the terminal cost is more conservative than in [10].The terminal cost is designed from the parameter-dependent Lyapunov function

$$\mathcal{V}_\xi(x) = x'\left(\sum_{i=1}^{l}[\xi]_i P_i\right)x = x'\mathcal{P}(\xi)x, \tag{5}$$

where $P_i > 0$, for all $i \in \mathbb{Z}_{[1,l]}$, but the associated stabilizing control law needs to be linear

$$u = Kx. \tag{6}$$

Given (1a), the following gives a design of (5), (6) for (3).

*Proposition 1:* Given system (1a), let $G, S_i \in \mathbb{R}^{n\times n}$, $S_i > 0$, $i \in \mathbb{Z}_{[1,l]}$, $E \in \mathbb{R}^{m\times n}$, be such that

$$\begin{bmatrix} G+G'-S_i & (A_iG+B_iE)' & E' & G' \\ A_iG+B_iE & S_j & 0 & 0 \\ E & 0 & R^{-1} & 0 \\ G & 0 & 0 & Q^{-1} \end{bmatrix} > 0 \tag{7}$$

for all $i, j \in \mathbb{Z}_{[1,l]}$. Then, $G$ is full rank, and $P_i = S_i^{-1}$, $i \in \mathbb{Z}_{[1,l]}$, $K = EG^{-1}$ satisfy

$$\begin{aligned}(\sum_{i=1}^{l}[\xi]_i(A_i+B_iK))'(\sum_{i=1}^{l}[\varsigma]_iP_i)(\sum_{i=1}^{l}[\xi]_i(A_i+B_iK)) \\ +Q+K'RK-\sum_{i=1}^{l}[\xi]_iP_i < 0,\end{aligned} \tag{8}$$

for any $\xi, \varsigma \in \Xi$. □

For the subsequent developments we assume that the following holds for (1a).

*Assumption 1:* For the given $A_i$, $B_i$, $i \in \mathbb{Z}_{[1,\ell]}$, $Q$, $R$, (7) admits a feasible solution. □

Assumption 1 is related to the existence of an (unconstrained, local) stabilizing linear control law for the uncertain system (1a), see, e.g., [4], [5], [12]. Indeed, if the uncertainty is too large, i.e., the vertex systems are excessively different, (7) may be infeasible, because a stabilizing controller for the uncertain system does not exist. Being solved at design time, the infeasibility of (7) will be recognized before controller execution and the system can be re-engineered, or a different control method can be chosen.

### *A. ISS with respect to Parameter Estimation Error*

First, we observe that the value function in (3) is Lipschitz-continuous in any bounded set. While such property may be inferred from the case of linear systems with known parameters, an explicit derivation for (1a) allows to obtain useful intermediate results that will be exploited later.

*Lemma 1:* Consider problem (3), where $\mathcal{X}_N = \mathcal{X} = \mathbb{R}^n$, $\mathcal{U} = \mathbb{R}^m$, $\mathcal{C}_{xu} = \mathbb{R}^{n+m}$, and $\mathcal{U} = \mathbb{R}^m$. For every $r \in \mathbb{R}_+$, there is $L \in \mathbb{R}_+$, such that for every $\xi^N \in \Xi^{N+1}$, the value function $\mathcal{V}^{MPC}_{\xi^N}(x)$ of (3), where $\mathcal{P}(\xi)$ is designed according to (5), is Lipschitz-continuous with constant $L$ in $B_r(0)$ for any finite $r \in \mathbb{R}_+$,

$$|\mathcal{V}^{MPC}_{\xi^N}(x_1) - \mathcal{V}^{MPC}_{\xi^N}(x_2)| \le L\|x_1 - x_2\|,\ \forall x_1, x_2 \in B_r(0). \tag{9}$$

*Proof (sketch):* Let $x_1, x_2 \in B_r(0)$, $U_t^* = [u^*_{0|t}\ \dots\ u^*_{N-1|t}]$ be the solution of (3) when $x_{0|t} = x_1$, and $x^1_{k|t}$, $k\mathbb{Z}_{[0,N]}$ be the corresponding the predicted state trajectory. Let $x^2_{k|t}$, $k \in [0, N]$ be the predicted state trajectory obtained by $U_t^*$ from $x_{0|t} = x_2$. From the value function one can show

$$\|x^2_{k|t} - x^1_{k|t}\| \le \max(\gamma_A, 1)^N\|x_2 - x_1\|,\ \mathbb{Z}_{[0,N]}, \tag{10}$$

where $\gamma_A := \max\{i \in \mathbb{Z}_{[1,l]} : \|A_i\|\}$.

It can be shown that $\|x^1_{k|t}\|$ is bounded for all $k \in \mathbb{Z}_{[0,N]}$. From the predicted unforced response $x^3_{k|t}$,

$$\mathcal{V}^{MPC}_{\xi^N}(x_1) \le (\gamma_P + N\|Q\|)(\max(\gamma_A, 1)^N\|x_1\|)^2. \tag{11}$$

where $\gamma_P := \max\{i \in \mathbb{Z}_{[1,l]} : \|P_i\|\}$, and $\|x^1_{k|t}\| \le \left(\frac{\gamma_P+N\|Q\|}{\lambda_{\min}(Q)}\right)^{1/2}(\max(\gamma_A, 1)^N)\|x_1\|$, $\|x^1_{N|t}\| \le \left(\frac{\gamma_P+N\|Q\|}{\vartheta_P}\right)^{1/2}(\max(\gamma_A, 1)^N)\|x_1\|$, where $\vartheta_P := \min\{i \in \mathbb{Z}_{[1,l]} : \lambda_{\min}(P_i)\}$. Thus,

$$\|x^1_{k|t}\| \le C_1\|x_1\|,\ k \in \mathbb{Z}_{[0,N]}, \tag{12}$$

where $C_1 = \left(\frac{\gamma_P+N\|Q\|}{\min\{\lambda_{\min}(Q),\gamma_{P_\lambda}\}}\right)^{1/2}(\max(\gamma_A, 1)^N)$.

For $x_1, x_2 \in B_r(0)$, $\|x^2_{k|t}\| \le C_1r + \max(\gamma_A, 1)^N 2r = C_2r$. Hence $\mathcal{V}^{MPC}_{\xi^N}(x_2) - \mathcal{V}^{MPC}_{\xi^N}(x_1) \le ((C_1 + C_2)r(\gamma_P + N\|Q\|)(\max(\gamma_A, 1)^N))\|x_2 - x_1\| = C_3\|x_2 - x_1\|$. The reverse inequality is shown in a similar way to conclude that for every $\xi^N \in \Xi^{N+1}$, $|\mathcal{V}^{MPC}_{\xi^N}(x_1) - \mathcal{V}^{MPC}_{\xi^N}(x_2)| \le L\|x_1 - x_2\|$ where $L = C_3$. ■

Next, we obtain the ISS property of IAMPC .

*Theorem 1:* Let Assumption 1 hold, and let $\mathcal{X}_L$ be any compact set in $\mathbb{R}^n$. For the IAMPC with parameter update (4), that at every step solves (3) where $\mathcal{P}(\xi)$ is designed according to (5) and (7), $\mathcal{X}_N = \mathcal{X} = \mathbb{R}^n$, $\mathcal{U} = \mathbb{R}^m$, $\mathcal{C}_{xu} = \mathbb{R}^{n+m}$, $\mathcal{U} = \mathbb{R}^m$, $\mathcal{V}^{\rm MPC}_{\xi^N_t}(x(t))$ is such that

$$\begin{aligned}\mathcal{V}^{MPC}_{\xi^N_{t+1}}(x(t+1)) - \mathcal{V}^{MPC}_{\xi^N_t}(x(t)) \le \\ -\lambda_{\min}(Q)\|x(t)\|^2 + \gamma_{ISS}\|\tilde{\xi}_{0|t}\|\end{aligned} \tag{13}$$

where $\gamma_{ISS} \in \mathbb{R}_+$. Thus, $\mathcal{V}^{\rm MPC}_{\xi^N}(x)$ is an ISS-Lyapunov function with respect to the estimation error $\tilde{\xi}_{0|t} = \bar{\xi} - \xi_{0|t} \in \tilde{\Xi}(\xi_{0|t})$ for (1) in closed loop with the IAMPC based on (3) in any $\mathcal{X}_\eta \subseteq \mathcal{X}_L$, where $\mathcal{X}_\eta$ is RPI with respect to $\tilde{\xi}_{0|t}$ for the closed loop.

*Proof (sketch):* The prediction error is $\|\varepsilon_x\| \le \gamma_A\|\tilde{\xi}_{0|t}\|_1\|x(t)\| + \gamma_B\|\tilde{\xi}_{0|t}\|_1\|u^*_{0|t}\|$, where $\gamma_A$ is defined in the proof of Lemma 1, and $\gamma_B = \max_{i=1,\dots l}\|B_i\|$.

By Lemma 1 we obtain

$$\begin{aligned}\mathcal{V}^{MPC}_{\xi^N_{t+1}}(x(t+1)) - \mathcal{V}^{MPC}_{\xi^N_t}(x(t)) \le -\lambda_{\min}(Q)\|x(t)\|^2 \\ + L(\gamma_A\|x(t)\| + \gamma_B\|u^*_{0|t}\|)\|\tilde{\xi}_{0|t}\|_1.\end{aligned} \tag{14}$$

and by the bounds on the value function we have $\|u^*_{k|t}\| \le \left(\frac{\gamma_P+N\|Q\|}{\lambda_{\min}(R)}\right)^{1/2}(\max(\gamma_A, 1)^N)\|x(t)\| = L_u\|x(t)\|$. Due to compactness of $\mathcal{X}_L$ , there exists $\gamma \in \mathbb{R}_+$ such that $\|x\| \le \gamma$, and, due to the norm equivalence in finite dimensional spaces, there exists $\gamma_p$ such that $\|\tilde{\xi}_{0|t}\|_1 \le \gamma_p\|\tilde{\xi}_{0|t}\|$.

Combining with (14), we conclude that $\mathcal{V}^{MPC}_{\xi^N_{t+1}}(x(t+1)) - \mathcal{V}^{MPC}_{\xi^N_t}(x(t)) \leq -\lambda_{\min}(Q)\|x(t)\|^2 + \gamma_{ISS}\|\tilde{\xi}_{0|t}\|$, with $\gamma_{ISS} = L(\gamma_A + L_u\gamma_B)\gamma_p\gamma$. ■

### *B. Asymptotic stability with bounded estimation error*

By the ISS result in Theorem 1, if eventually the estimation error vanishes, i.e., $\tilde{\xi}(t) = 0$ for all $t \geq \tau$, the closed loop of (1a) with IAMPC is AS. Next, we show that $\mathcal{V}^{MPC}_{\xi^N_t}(x(t))$ is a Lyapunov function for the closed-loop system when $\|\tilde{\xi}_{0|t}\|$ is sufficiently small, and therefore the closed loop is AS even in presence of a small, yet non-zero, estimation error.

*Theorem 2:* Given system (1) with unknown parameter $\bar{\xi}$, let the assumptions of Theorem 1 hold, and let $u(t) = u^*_{0|t}$ be determined by the IAMPC with parameter update (4) that at every step solves (3), where $\mathcal{P}(\xi)$ is designed according to (5), $\mathcal{X}_N = \mathcal{X} = \mathbb{R}^n$, $\mathcal{U} = \mathbb{R}^m$, $\mathcal{C}_{xu} = \mathbb{R}^{n+m}$. There exists $\delta > 0$ such that if $\|\tilde{\xi}_{0|t}\|_1 \leq \Delta < \delta$ for some $\Delta > 0$, for all $t \in \mathbb{Z}_{0+}$, the closed loop is AS.

*Proof (sketch):* For any $x_1, x_2 \in \mathbb{R}^n$, $\|x^1_{k|t} + x^2_{k|t}\| \leq 2C_1\|x_1\| + \max(\gamma_A, 1)^N\|x_2 - x_1\|$, for $k \in \mathbb{Z}_{[0,N]}$. Hence,

$$\begin{aligned}\mathcal{V}^{MPC}_{\xi^N}(x_2) - \mathcal{V}^{MPC}_{\xi^N}(x_1) &\leq (\gamma_P + N\|Q\|)(2C_1\|x_1\| \\ &\quad + \max(\gamma_A, 1)^N\|x_2 - x_1\|) \cdot \max(\gamma_A, 1)^N\|x_2 - x_1\| \\ &:= C_4\|x_1\| \cdot \|x_2 - x_1\| + C_5\|x_2 - x_1\|^2. \end{aligned} \tag{15}$$

From (15), $\mathcal{V}^{MPC}_{\xi^N}(x(t+1)) - \mathcal{V}^{MPC}_{\xi^N}(x_{1|t}) = C_4\|x_{1|t}\| \cdot \|\varepsilon_x\| + C_5\|\varepsilon_x\|^2$. From the bounds on $\varepsilon_x$ with $C_6 := L_u$.

$$\begin{aligned}\mathcal{V}^{MPC}_{\xi^N}(x(t+1)) - \mathcal{V}^{MPC}_{\xi^N}(x_{1|t}) &\leq (C_4C_1(\gamma_A + \gamma_B C_6) \\ &\quad + 2C_5(\gamma_A + \gamma_B C_6)^2)\|\tilde{\xi}_{0|t}\|_1\|x(t)\|^2 \end{aligned} \tag{16}$$

Let

$$\|\tilde{\xi}_{0|t}\|_1 \leq \Delta < \frac{\lambda_{\min}(Q)}{C_4C_1(\gamma_A + \gamma_B C_6) + 2C_5(\gamma_A + \gamma_B C_6)^2}, \tag{17}$$

then $\mathcal{V}^{MPC}_{\xi^N_{t+1}}(x(t+1)) - \mathcal{V}^{MPC}_{\xi^N_t}(x(t)) \leq -\epsilon\|x(t)\|^2$, hence the origin of the closed-loop system is AS. ■

The next result follows directly from Theorem 2 and [10].

*Result 2:* IAMPC based on (3), (4), where $\mathcal{P}(\xi)$ is designed according to (5), achieves $(i) - (iv)$ in Problem 1. □

Based on Theorem 2, an alternative ISS Lyapunov function can be constructed for the closed-loop system.

*Corollary 1:* Given (1), let the assumptions of Theorem 1 hold. For any $\alpha \in (0, 1)$, there exists $\Delta(\alpha) \in \mathbb{R}_+$ such that

$$\begin{aligned}\mathcal{V}^{MPC}_{\xi^N_{t+1}}(x(t+1)) - \mathcal{V}^{MPC}_{\xi^N_t}(x(t)) &\leq -\alpha \cdot \lambda_{\min}(Q)\|x(t)\|^2 \\ &\quad + \gamma'_{ISS}\max\{0, \|\tilde{\xi}_{0|t}\|_1 - \Delta(\alpha)\} \end{aligned} \tag{18}$$

where $\gamma'_{ISS} \in \mathbb{R}_+$. □

## IV. CONSTRAINED IAMPC: ROBUST CONSTRAINT DESIGN AND STABILITY RESULTS

Next we consider the case when (1) is subject to constraints, i.e., $\mathcal{X} \times \mathcal{U} \subset \mathbb{R}^n \times \mathbb{R}^m$, and make the following assumption on the constraint sets.

*Assumption 2:* $\mathcal{X}$, $\mathcal{U}$ are compact polyhedra with $0 \in \mathrm{int}(\mathcal{X})$, $0 \in \mathrm{int}(\mathcal{U})$. □

For achieving recursive feasibility and stability properties in presence of constraints, appropriate designs for the terminal set $\mathcal{X}_N$ and the robust constraint set $\mathcal{C}_{xu}$ are needed. For $\mathcal{C}_{xu}$, the maximal RCI set for (1a) was used in [10]. Such design achieves the least restrictive set $\mathcal{C}_{xu}$, which however can be arbitrarily complex in terms of number of describing inequalities, and hence very expensive to compute and use. Further, using the maximal RCI imposes a lower bound on the prediction horizon $N \in \mathbb{Z}_+$ in the MPC problem (3).

Here, by exploiting the LMI (7) we propose an alternative based on constructing the $N$-step backward reachable set of a specific RPI set. Let $\mathcal{X}_{xu} = \mathcal{X} \times \mathcal{U}$ be a set of feasible states and inputs with $0 \in \mathrm{int}(\mathcal{X}_{xu})$. Let $K$ be determined from Proposition 1 and construct the set sequence

$$\begin{aligned}\mathcal{X}^{(0)} &= \{x : \ (x, Kx) \in \mathcal{X}_{xu}\} \\ \mathcal{X}^{(h+1)} &= \{x : \ (A_i + B_iK)x \in \mathcal{X}^{(h)}, \ \forall i \in \mathbb{Z}_{[1,\ell]}\} \cap \mathcal{X}^{(h)} \\ \mathcal{X}^{\infty} &= \lim_{h\to\infty} \mathcal{X}^{(h)}. \end{aligned} \tag{19}$$

In [13] it was proved that the sequence in (19) reaches a fixpoint in a finite number of steps, i.e., there exists a finite $\bar{h} \in \mathbb{Z}_{0+}$ such that $\mathcal{X}^{(\bar{h}+1)} = \mathcal{X}^{(\bar{h})} = \mathcal{X}^{\infty}$.

*Lemma 2:* Consider (1), for which (6) is computed from (7), and $\mathcal{X}^{\infty}$ is computed from (19) with $\mathcal{X}_{xu} = \mathcal{X} \times \mathcal{U}$. Then, $\mathcal{X}^{\infty}$ is RPI for (1) in closed loop with (6) for every $\tilde{\xi}_{0|t} \in \tilde{\Xi}(\xi_{0|t})$. □

The proof of Lemma (2) follows from the definition and the convexity of $\mathcal{X}_{\infty}$, and from using $u = Kx$.

$\mathcal{X}^{\infty}$ is RPI for (1) in closed loop with (6) for every $\tilde{\xi}_{0|t} \in \tilde{\Xi}(\xi_{0|t})$ because (6) is a linear feedback, as opposed to the parameter-dependent linear feedback in [10]. Using a parameter-dependent feedback does not guarantee, in general, robustness in the case of errors in the parameter estimate, i.e., if the parameter used for the input computation is different from the one in the actual system dynamics. However, since here we restrict ourselves to use (6), such an issue cannot occur.

We construct an RCI set for (1), from the RPI set $\mathcal{X}^{\infty}$. Compute the robust backward reachable set sequence

$$\begin{aligned}\mathcal{R}^{(0)} &= \mathcal{X}^{\infty}, \\ \mathcal{R}^{(h+1)} &= \{x \in \mathcal{X} : \ \exists u \in \mathcal{U}, A_ix + B_iu \in \mathcal{R}^{(h)}, \forall i \in \mathbb{Z}_{[1,\ell]}\}, \end{aligned} \tag{20}$$

where $\mathcal{R}^{(h)}$ is the set of states that can be brought to $\mathcal{R}^{(0)}$ in $h$ steps by using state feedback, while satisfying state and input constraints, for any unknown $\xi^h \in \Xi^h$ and any $\tilde{\xi}^h \in \tilde{\Xi}^h(\xi^h)$. Due to $\mathcal{R}^{(0)}$ being RPI, $\mathcal{R}^{(h+1)} \supseteq \mathcal{R}^{(h)}$ and $\mathcal{R}^{(h)}$ is RCI for every $h \in \mathbb{Z}_{0+}$.

From $\mathcal{X}^{\infty}$ and $\{\mathcal{R}^{(h)}\}_h$ we design the terminal set $\mathcal{X}_N$ and the robust constraint set $\mathcal{C}_{xu}$ in (3) as

$$\mathcal{X}_N = \mathcal{X}^{\infty}, \tag{21}$$

$$\begin{aligned}\mathcal{C}_{xu} = &\{(x, u) : x \in \mathcal{R}^{(N)}, u \in \mathcal{U}, \\ &A_ix + B_iu \in \mathcal{R}^{(N)}, \ \forall i \in \mathbb{Z}_{[1,\ell]}\}, \end{aligned} \tag{22}$$

respectively, that are shown next to achieve $(ii)$ in Problem 1.

*Lemma 3:* Consider (3), (6) computed from (7), and $\mathcal{X}_N = \mathcal{X}^\infty$ computed from (19) with $\mathcal{X}_{xu} = \mathcal{X} \times \mathcal{U}$. Given $N \in \mathbb{Z}_+$, let $\mathcal{C}_{xu}$ in (3e) be defined by (22) with $\mathcal{R}^{(N)}$ defined in (20). If $x(t) \in \mathcal{R}^{(N)}$ at $t \in \mathbb{Z}_{0+}$, and $\xi_\tau^N \in \Xi^{N+1}$, $\tilde{\xi}_{0|\tau} \in \tilde{\Xi}(\xi_{0|\tau})$ for all $\tau \geq t$, (3) is feasible for all $\tau \geq t$. $\square$

The proof of Lemma (3) follows from the RCI properties, and $\mathcal{R}^{(N)}$ being the $N$-steps backward reachable set of $\mathcal{X}_N$.

When compared to the design based on the maximal RCI set proposed in [10], the advantages of the design for $\mathcal{C}_{xu}$ proposed here are that the prediction horizon $N$ is a free design variable, and, since all the sets $\mathcal{R}^{(h)}$ are RCI, $N$ can be chosen to control the complexity of $\mathcal{C}_{xu}$, which is finite for any finite $N$ [11]. On the other hand $\mathcal{C}_{xu}$ will be in general smaller than the maximal RCI, thus reducing the domain of attraction of the closed loop.

### A. Asymptotic stability with bounded estimation error

*Theorem 3:* Given system (1) with unknown parameter $\bar{\xi}$, let Assumptions 1 and 2 hold, and given $N \in \mathbb{Z}_+$, consider the optimal control problem (3). Let $\mathcal{P}(\xi)$ be designed according to (5), $\mathcal{X}_N, \mathcal{C}_{xu}$ be determined by (21), (22), respectively. Let $u(t) = u^*_{0|t}$ be determined by the IAMPC that at every step solves (3) with parameter update (4). If $x(t) \in \mathcal{R}^{(N)}$ at $t \in \mathbb{Z}_{0+}$, then the constraints (1b) are satisfied for all $\tau \geq t$. Furthermore, there exists $\bar{\delta} > 0$ such that if $\|\tilde{\xi}_{0|\tau}\|_1 \leq \Delta < \bar{\delta}$ for all $\tau \geq t$, for some $\Delta > 0$, then the origin of the closed-loop system is AS.

*Proof (sketch):* The proof follows the lines of that of Theorem 2. We consider again the predicted state sequences $x^2_{k|t}$, $x^2_{k|t}$ from the proof of Lemma 1 which here need also to satisfy $(i)$, $x^2_{k|t} \in \mathcal{X}, x^3_{k|t} \in \mathcal{X}$, for all $k \in \mathbb{Z}_{[0,N-1]}$, $(ii)$, $x^2_{N|t} \in \mathcal{X}_N, x^3_{N|t} \in \mathcal{X}_N$, $(x^2_{k|t}, u^*_{k|t}) \in \mathcal{C}_{xu}, (x^3_{k|t}, 0) \in \mathcal{C}_{xu}, k \in \mathbb{Z}_{[0,N-1]}$. Note that, by construction, $0 \in \text{int}(\mathcal{C}_{xu})$.

By the properties of $\mathcal{X}_\infty$, there exists $r_1 > 0$ such that $B_{r_1}(0) \subset \mathcal{X}_N \subset \mathcal{X}$. From the intermediate steps of Lemma 1 if $\max(\gamma_A, 1)^N \|x_1\| < r_1$, then $x^3_{k|t} \in \mathcal{X}$ for $k \in \mathbb{Z}_{[0,N-1]}$, $x^3_{N|t} \in \mathcal{X}_N$. From $\|x^2_{k|t}\| \leq C_1\|x_1\| + \max(\gamma_A, 1)^N \|x_2 - x_1\|$, if $C_1\|x_1\| < r_1/2$ and $\max(\gamma_A, 1)^N\|x_2 - x_1\| < r_1/2$, $x^2_{k|t} \in \mathcal{X}$ for $k \in \mathbb{Z}_{[0,N-1]}$, and $x^2_{N|t} \in \mathcal{X}_N$. Thus $(i), (ii)$ hold locally.

Conditions for $(iii)$ to hold are found similarly, using the fact, from previous proofs, $\|(x^2_{k|t}, u^*_{k|t})\| \leq (C_1\|[I\ 0]'\| + C_6\|[0\ \ I]'\|)\|x_1\| + \|[I\ \ 0]'\| \max(\gamma_A, 1)^N \|x_2 - x_1\|$. Since $0 \in \text{int}(\mathcal{C}_{xu})$, there exists $r_2 > 0$ such that $B_{r_2}(0) \subset \mathcal{C}_{xu}$. If $(C_1\|[I\ 0]'\| + C_6\|[0\ I]'\|)\|x_1\| < r_2/2$, and $\|[I\ \ 0]'\| \max(\gamma_A, 1)^N \|x_2 - x_1\| < r_2/2$ then $(x^2_{k|t}, u^*_{k|t}) \in \mathcal{C}_{xu}$ for $k \in \mathbb{Z}_{[0,N-1]}$. Similarly, if $\|[I\ \ 0]'\| \max(\gamma_A, 1)^N \|x_1\| < r_2$, then $(x^3_{k|t}, 0) \in \mathcal{C}_{xu}$ for $k \in \mathbb{Z}_{[0,N-1]}$. Combining the above, there exist $\bar{r} > 0$, $\bar{\varepsilon} > 0$ such that if $\|x_1\| < \bar{r}$, $\|x_2 - x_1\| < \bar{\varepsilon}$, then $(iii)$ is satisfied. In particular, $\bar{r} = \min\{\frac{r_1}{\max(\gamma_A,1)^N}, \frac{r_2}{2(C_1\|[I\ 0]'\|+C_6\|[0\ I]'\|)}, \frac{r_2}{\|[I\ 0]'\| \max(\gamma_A,1)^N}, \frac{r_1}{2C_1}\}$, and $\bar{\varepsilon} = \min\{\frac{r_1}{2\max(\gamma_A,1)^N}, \frac{r_2}{2\|[I\ 0]'\| \max(\gamma_A,1)^N}\}$.

If $\|x(t)\| < \min\{\bar{r}, \frac{\bar{r}}{C_1}\} := \tilde{r}$ and $\|\tilde{\xi}_{0|t}\|_1 \leq (C_1\bar{\varepsilon})/((\gamma_A + \gamma_B C_6)\bar{r})$, then $\mathcal{V}^{MPC}_{\xi^N_{t+1}}(x(t+1)) - \mathcal{V}^{MPC}_{\xi^N_{t+1}}(x_{1|t}) \leq C_4\|x_{1|t}\| \cdot \|\varepsilon_x\| + C_5\|\varepsilon_x\|^2$, and with an argument similar to that for Theorem 2 there is $\Delta > 0$ such that if $\|x(t)\| < \tilde{r}$, and $\|\tilde{\xi}_{0|t}\|_1 \leq \Delta < \bar{\delta}$ then

$$\mathcal{V}^{MPC}_{\xi^N_{t+1}}(x(t+1)) - \mathcal{V}^{MPC}_{\xi^N_t}(x(t)) \leq -\epsilon\|x(t)\|^2,$$

for some $\epsilon > 0$, thus, AS is proved. Here $\bar{\delta} = \min\{\frac{C_1\bar{\varepsilon}}{(\gamma_A+\gamma_B C_6)\bar{r}},\ (\lambda_{\min}(Q))/(C_4C_1(\gamma_A + \gamma_B C_6) + 2C_5(\gamma_A + \gamma_B C_6)^2)\}$. $\blacksquare$

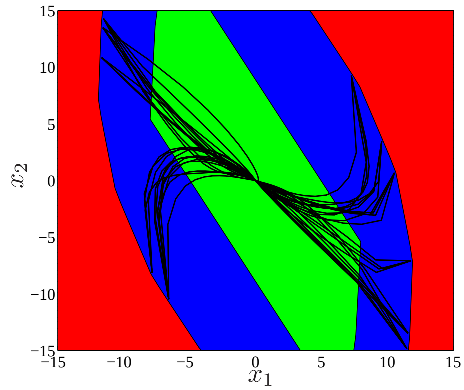


Fig. 1. Simulation of the IAMPC in closed loop with the numerical example with $\mathcal{R}^{(N)}$, $N = 8$, phase plane trajectories (black), $\mathcal{X}^\infty$ (green), $\mathcal{R}^{(N)}$ (blue), $\mathcal{X}$ (red).

The next result follows directly from Theorem 3 and [10].

*Result 3:* IAMPC based on (3), (4), where $\mathcal{P}(\xi)$ is designed according to (5) and $\mathcal{X}_N, \mathcal{C}_{xu}$ are determined by (21), (22), achieves $(iv)$ in Problem 1. $\square$

## V. Numerical Simulations

*Example 1:* We consider (1), where $\ell = 10$, and the vertex matrices are: $A_{1+5i} = \left[\begin{smallmatrix} 1 & 0.2 \\ 0 & 1 \end{smallmatrix}\right]$, $A_{2+5i} = 1.1 \cdot A_{1+5i}$, $A_{3+5i} = 0.6 \cdot A_{1+5i}$, $A_{4+5i} = \left[\begin{smallmatrix} 0.9 & 0.3 \\ 0.4 & 0.6 \end{smallmatrix}\right]$ $A_{5+5i} = \left[\begin{smallmatrix} 0.95 & 0 \\ 0.8 & 1.02 \end{smallmatrix}\right]$, $i \in \{0, 1\}$ and $B_j = \left[\begin{smallmatrix} -0.035 & -0.905 \end{smallmatrix}\right]'$, $B_{j+5} = 0.9 \cdot B_1$, $j \in \mathbb{Z}_{[1,5]}$. While being only of $2^{nd}$ order, the challenges in this example are in some of the vertex systems being unstable, and some vertex matrices being significantly different from the others. For a similar system with uncertainty only in $A$, it was shown in [13] that without proper cost adaptation, the closed-loop may not be AS, even if the perfect model was estimated. The constraints are defined by (1b), where $\mathcal{X} = \{x \in \mathbb{R}^2 : |[x]_i| \leq 15,\ i = 1, 2\}$, $\mathcal{U} = \{u \in \mathbb{R} :\ |[u]| \leq 10\}$.

We have implemented a simple estimator that computes the least squares solution $\varrho(t)$ based on past data window of $N_m$ steps and applies a first order filter on the projection of $\varrho(t)$ onto $\Xi$, i.e., $\xi(t+1) = (1 - \varsigma)\xi(t) + \varsigma \cdot \text{proj}_\Xi(\varrho(t))$, where $\varsigma \in \mathbb{R}_{(0,1)}$, and $[\xi(0)]_i = 1/\ell$, $i \in \mathbb{Z}_{[1,\ell]}$. Such simple estimator guarantees that $\xi(t) \in \Xi$ for all $t \in \mathbb{Z}_{0+}$ because projection and summation guarantee that the result is a convex combination vector. We set $\varsigma = 1/8$, $N_m = 5$.

We design the controller according to Theorem 3, where we impose $N = 8$, where $\mathcal{R}^{(N)}$ is determined by (20). Figure 1 shows the simulations where the initial condition lies within $\mathcal{R}^{(N)}$ and for each initial condition, 4 simulations with different (random) values of $\bar{\xi} \in \Xi$ are executed.

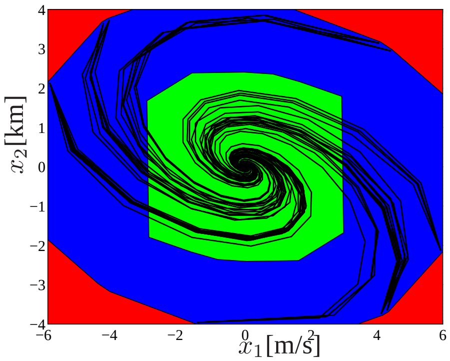


Fig. 2. Simulation of the IAMPC in closed loop with the satellite example with $\mathcal{R}^{(N)}$, $N = 8$, phase plane trajectories (black), $\mathcal{X}^{\infty}$ (green), $\mathcal{R}^{(N)}$ (blue), $\mathcal{X}$ (red).

*Example 2:* We consider the out-of-orbital-plane dynamics of a satellite, which are naturally decoupled from the in-orbital-plane dynamics, and are subject to significant perturbations for orbits within Earth's orbital plane [15]. Hence, the satellite may drift "far" from its desired value along such axis. By the HCW equations of relative motion [15], the out-of-orbital-plane (z-axis in HCW) dynamics are

$$\dot{x}(t) = \begin{bmatrix} 0 & -n^2 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} \frac{1}{m} \\ 0 \end{bmatrix} u(t) \tag{23}$$

where $[x]_1$ is the $z$-axis velocity [m/s], $[x]_2$ is the $z$-axis position [km], $u$ is the $z$-axis thrust [N], $n$ is the orbital frequency and $m$ is the mass of the satellite. We consider uncertainty in the orbital frequency, i.e., the orbit, and satellite mass. In particular, the nominal and vertex values of $n$ and $m$ are : $n_0 = 9.5 \times 10^{-4}, n_1 = 0.8 \cdot n_1, n_2 = 1.2 \cdot n_1$; $m_0 = 2000, m_1 = 0.75 \cdot m_1, m_2 = 1.25 \cdot m_1$, where the nominal value $n_0$ is in low earth orbit (LEO). The dynamics are formulated in discrete time in the form of (1) with a sampling period of $T_s = 300$ seconds, and $\ell = 4$. The constraints are defined by (1b), where $\mathcal{X} = \{x \in \mathbb{R}^2 : |[x]_1| \leq 6\ \mathrm{m/s},\ |[x]_2| \leq 4\ \mathrm{km}\}, \mathcal{U} = \{u \in \mathbb{R} :\ |u| \leq 50\ \mathrm{N}\}$.

We design the controller according to Theorem 3, where we impose $N = 8$, where $\mathcal{R}^{(N)}$ is determined by (20). We use the same parameter estimator as in Example 1, now with $N_m = 3$, $\varsigma = 1/16$, which are expected to give slower convergence. Figure 2 and Figure 3 report the simulation results, showing both constraint satisfaction and stabilization.

## VI. Conclusions and Future Work

We have extended the recently developed IAMPC method [10] to account for uncertainty in the input matrix ($B$), and we have provided additional design procedures and stability results. We have shown that the closed-loop is AS even for small, yet non-zero, parameter estimator errors, which also allowed to derive a tighter ISS Lypaunov function. We have also proposed an alternative method to design constraints based on constructing a RCI set as the $N$-steps backward reachable set of the terminal set, which allows to maintain the MPC prediction horizon as a free design choice, it is in general faster to compute, and has a reduced complexity. In the future we plan to extend the proposed method to tracking problems and to consider the case of partial state information.

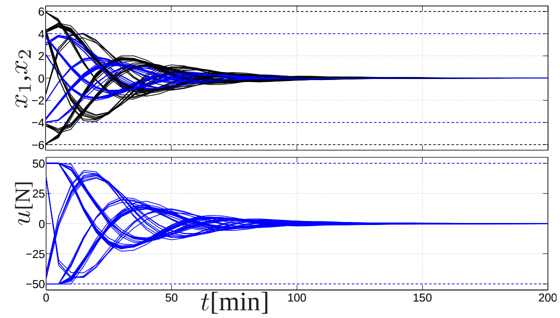


Fig. 3. Simulations of input (bottom) and state (top) trajectories against time, $[x]_1$ blue, $[x]_2$ black. Constraints as dash lines

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