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TR2014-085 August 2014

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World Congress of the International Federation of Automatic Control (IFAC)

Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Extremum Seeking-based Indirect Adaptive Control for Nonlinear Systems

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Abstract: We present in this paper a preliminary result on extremum seeking (ES)-based adaptive trajectory tracking control for nonlinear systems. We propose, for the class of nonlinear systems with parametric uncertainties which can be rendered integral Input-to-State stable (iISS) w.r.t. the parameter estimation errors input, that it is possible to merge together the integral Input-to-State stabilizing feedback controller and a model-free extremum seeking algorithm to realize a learning-based indirect adaptive controller. We show the efficiency of this approach on a mechatronic example.

1. INTRODUCTION

Classical adaptive control deals with controlling partially unknown process based on their uncertain model, i.e., controlling plants with parameters uncertainties. Classical adaptive methods can be classified as 'direct', where the controller is updated to adapt to the process, or 'indirect', where the model is updated to better reflect the actual process. Many adaptive methods have been proposed over the years for linear and nonlinear systems, we could not possibly cite here all the design and analysis results that have been reported, instead we refer the reader to e.g. Landau et al. [2011], Krstic et al. [1995] and the references therein for more details. What we want to underline here is that these results in 'classical' adaptive control are mainly based on the structure of the model of the system, e.g. linear vs. nonlinear, with linear uncertainties parametrization vs. nonlinear parameterizations, etc.

On the other hand, Extremum seeking (ES) is a well known approach by which one can search for the extremum of a cost function associated with a given process performance (under some conditions) without the need for a detailed model of the process, e.g. Ariyur and Krstić [2003], Ariyur and Krstic [2002], Nesic [2009]. Several ES algorithms with their stability analysis have been proposed, e.g. Scheinker [2013], Krstic [2000], Ariyur and Krstic [2002], Tan et al. [2006], Nesic [2009], Tan et al. [2006], Ariyur and Krstić [2003], Rotea [2000], Guay et al. [2013], and many applications of ES algorithms have been reported, e.g Zhang et al. [2003], Hudon et al. [2008], Zhang and Ordez [2012], Benosman and Atinc [2013a,c].

Another worth mentioning paradigm is the one which uses 'learning schemes' to estimate the uncertain part of the process. Indeed, in this paradigm the learning-based controller, based either on machine learning theory, neural network, fuzzy systems, etc. is trying either to estimate the parameters of an uncertain model, or the structure of a deterministic or a stochastic function representing part or totality of the model. Several results have been proposed in this area as well, and we refer the reader to e.g. Wang and Hill [2006] and the references therein for more details. We want to concentrate in this paper on the use of ES theory in the 'learning-based' adaptive control paradigm. Indeed, several results were recently developed in this direction, e.g. Haghi and Ariyur [2011], Ariyur et al. [2009], Guay and Zhang [2003], Adetola and Guay [2007], Zhang et al. [2003], Hudon et al. [2008], Benosman and Atinc [2013a,c]. For instance in Haghi and Ariyur [2011], Ariyur et al. [2009] the authors used a model-free ES, i.e., only based on a desired cost function, to estimate parameters of a linear state feedback to compensate for unknown parameters for linear systems. In Guay and Zhang [2003], Adetola and Guay [2007] an extremum seeking-based controller for nonlinear affine systems with linear parameters uncertainties was proposed. The controller drives the states of the system to unknown optimal states that optimize a desired objective function. The ES controller is not model-free in the sense that it is based on the known part of the model, i.e., it is designed based on the objective function and the nonlinear model structure. Similar approach is used in Zhang et al. [2003], Hudon et al. [2008] when dealing with more specific examples. In Benosman and Atinc [2013a], the authors used, for the case of electromagnetic actuators, a model-free ES, i.e., only based on the cost function without the use of the system model, to learn the 'best' feedback gains of a passive robust state feedback. Similarly, in Benosman and Atinc [2013c], a backstepping controller was merged with a model-free ES to estimate the uncertain parameters of a nonlinear model for electromagnetic actuators. Although, no stability analysis was presented for the full controller (i.e., backstepping plus ES estimator), very promising numerical results where reported.

In this work we propose to generalize the idea of Benosman and Atinc [2013c], for the class of nonlinear system with parametric uncertainties which can be rendered iISS w.r.t. the parameters estimation error. The idea is based on a modular design, where we first design a feedback controller which makes the closed-loop tracking error dynamic iISS w.r.t. the estimation errors and then complement this iISS-controller with a mode-free ES algorithm that can minimize a desired cost function, by tuning, i.e., estimating, the unknown parameters of the model. The modular design simplifies the analysis of the total controller, i.e., iISS-controller plus ES estimation algorithm. We first propose this formulation in the general case of nonlinear systems and then show a detailed case-study on a mechatronic example.

This paper is organized as follows: Section II is used to recall some notations and definitions. In Section III we present the main result of this paper, namely, the ES-based learning adaptive controller. Section IV is dedicated to an application example, and the paper ends with a Conclusion in Section V.

2. PRELIMINARIES

Throughout the paper we will use $\|.\|$ to denote the Euclidean norm; i.e., for $x \in \mathbb{R}^n$ we have $\|x\| = \sqrt{x^T x}$. We will use the notation $|.|$ for the absolute value of a scalar variable, and $\dot{(.)}$ for the short notation of time derivative. We denote by C^k functions that are k times differentiable. A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. A continuous function $\beta : [0, a) \times [0, \infty) \to [0, \infty)$ is said to belong to class $\mathcal{KL}$ if, for each fixed s, the mapping $\beta(r, s)$ belongs to class $\mathcal{K}$ with respect to r and, for each fixed r, the mapping $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \to 0$ as $s \to \infty$.
Let us now introduce some useful definitions.

Definition. 1 [Local Integral Input-to-State Stability Ito and Jiang [2009]]
Consider the system

$$\dot{x} = f(t, x, u) \tag{1}$$

where $x \in \mathcal{D} \subseteq \mathbb{R}^n$ such that $0 \in \mathcal{D}$, and $f : [0, \infty) \times \mathcal{D} \times \mathcal{D}_u \to \mathbb{R}^n$ is piecewise continuous in t and locally Lipschitz in x and u, uniformly in t. The inputs are assumed to be measurable and locally bounded functions $u : \mathbb{R}_{\geq 0} \to \mathcal{D}_u \subseteq \mathbb{R}^m$. Given any control $u \in \mathcal{D}_u$ and any $\xi \in \mathcal{D}_0 \subseteq \mathcal{D}$, there is a unique maximal solution of the initial value problem $\dot{x} = f(t, x, u)$, $x(t_0) = \xi$. Without loss of generality, assume $t_0 = 0$. The unique solution is defined on some maximal open interval, and it is denoted by $x(\cdot, \xi, u)$. System (1) is locally integral input-to-state stable (LiISS) if there exist functions $\alpha, \gamma \in \mathcal{K}$ and $\beta \in \mathcal{KL}$ such that, for all $\xi \in \mathcal{D}_0$ and all $u \in \mathcal{D}_u$, the solution $x(t, \xi, u)$ is defined for all $t \geq 0$ and

$$\alpha(\|x(t, \xi, u)\|) \leq \beta(\|\xi\|, t) + \int_0^t \gamma(\|u(s)\|) ds \tag{2}$$

for all $t \geq 0$. Equivalently, system (1) is LiISS if and only if there exist functions $\beta \in \mathcal{KL}$ and $\gamma_1, \gamma_2 \in \mathcal{K}$ such that

$$\|x(t, \xi, u)\| \leq \beta(\|\xi\|, t) + \gamma_1 \left(\int_0^t \gamma_2(\|u(s)\|) ds \right) \tag{3}$$

for all $t \geq 0$, all $\xi \in \mathcal{D}_0$ and all $u \in \mathcal{D}_u$. Note that if system (1) is LiISS, then the 0-input system is locally uniformly asymptotically stable (0-LUAS), that is, the unforced system

$$\dot{x} = f(t, x, 0) \tag{4}$$

is LUAS (Sontag and Wang [1996]).

Definition. 2 [ϵ- Semi-global practical uniform ultimate boundedness with ultimate bound δ (($\epsilon - \delta$)-SPUUB) Scheinker [2013]]
Consider the system

$$\dot{x} = f^{\epsilon}(t, x) \tag{5}$$

with $\phi^{\epsilon}(t, t_0, x_0)$ being the solution of (5) starting from the initial condition $x(t_0) = x_0$. Then, the origin of (5) is said to be (ϵ, δ)-SPUUB if it satisfies the following three conditions:
1-(ϵ, δ)-*Uniform Stability:* For every $c_2 \in]\delta, \infty[$, there exists $c_1 \in]0, \infty[$ and $\hat{\epsilon} \in]0, \infty[$ such that for all $t_0 \in \mathbb{R}$ and for all $x_0 \in \mathbb{R}^n$ with $||x_0|| < c_1$ and for all $\epsilon \in]0, \hat{\epsilon}[$,

$$||\phi^{\epsilon}(t, t_0, x_0)|| < c_2, \ \forall t \in [t_0, \infty[$$

2-(ϵ, δ)-*Uniform ultimate boundedness:* For every $c_1 \in]0, \infty[$ there exists $c_2 \in]\delta, \infty[$ and $\hat{\epsilon} \in]0, \infty[$ such that for all $t_0 \in \mathbb{R}$ and for all $x_0 \in \mathbb{R}^n$ with $||x_0|| < c_1$ and for all $\epsilon \in]0, \hat{\epsilon}[$,

$$||\phi^{\epsilon}(t, t_0, x_0)|| < c_2, \ \forall t \in [t_0, \infty[$$

3-(ϵ, δ)-*Global uniform attractivity:* For all $c_1, c_2 \in (\delta, \infty)$ there exists $T \in]0, \infty[$ and $\hat{\epsilon} \in]0, \infty[$ such that for all $t_0 \in \mathbb{R}$ and for all $x_0 \in \mathbb{R}^n$ with $||x_0|| < c_1$ and for all $\epsilon \in]0, \hat{\epsilon}[$,

$$||\phi^{\epsilon}(t, t_0, x_0)|| < c_2, \ \forall t \in [t_0 + T, \infty[$$

3. LEARNING-BASED ADAPTIVE CONTROLLER

Consider the system (1), with parametric uncertainties $\Delta \in \mathbb{R}^p$

$$\dot{x} = f(t, x, \Delta, u) \tag{6}$$

We associate with (6), the output vector

$$y = h(x) \tag{7}$$

where $h : \mathbb{R}^n \to \mathbb{R}^h$.
The control objective here is for y to asymptotically track a desired smooth vector time-dependent trajectory $y_{ref} : [0, \infty) \to \mathbb{R}^h$.
Let us now define the output tracking error vector as $e_y(t) = y(t) - y_{ref}(t)$.
We then assume the following
Assumption 1. There exists a robust control feedback $u_{iss}(t, x, \hat{\Delta}) : \mathbb{R}^{\times}\mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^m$, with $\hat{\Delta}(t)$ being the dynamic estimate of the uncertain vector Δ, such that, the closed-loop error dynamics

$$\dot{e}_y = f(t, e_y, e_{\Delta}) \tag{8}$$

is iISS from the input vector $e_{\Delta} = \Delta - \hat{\Delta}(t)$ to the state vector e_y.
Remark 2. Assumption 1 might seem too general, however, several control approaches can be used to design a controller u_{iss} rendering an uncertain system iISS, for instance backstepping control approach has been shown to achieve such a property for parametric strict-feedback systems, e.g. Krstic et al. [1995]. This is a preliminary report, and we do not pretend here to present a detailed solution for all the cases. A more detailed study of how to achieve Assumption 1 for specific classes of systems and how to use it in the context of ES learning-based adaptive control, will be presented in our future reports.

Let us define now the following cost function

$$Q(\hat{\Delta}, t) = F(e_y(\hat{\Delta}), t) \tag{9}$$

where $F : \mathbb{R}^h \times \mathbb{R}^+ \to \mathbb{R}^+$, $F(0, t) = 0$, $F(e_y, t) > 0$ for $e_y \neq 0$. We need the following assumptions on Q.
Assumption 3. The cost function Q has a local minimum at $\hat{\Delta}^* = \Delta$.
Assumption 4. $|\frac{\partial Q(\hat{\Delta}, t)}{\partial t}| < \rho_Q, \ \forall t \in \mathbb{R}^+, \ \forall \hat{\Delta} \in \mathbb{R}^p.$
Remark 5. Assumption 3 simply means that we can consider that Q has at least a local minimum at the true values of the uncertain parameters.

We can now present the following Lemma.
Lemma 6. Consider the system (6), (7), with the cost function (9), then under Assumptions 1, 3 and 4 , the controller u_{iss}, where $\hat{\Delta}$ is estimated with the multi-parameter extremum seeking algorithm

$$\dot{\hat{\Delta}}_i = a\sqrt{(}\omega_i) cos(\omega_i t) - k\sqrt{\omega_i} sin(\omega_i t) Q(\hat{\Delta}), \ i \in \{1, ..., p\} \tag{10}$$

with $a > 0$, $k > 0$, $\omega_i \neq \omega_j$, $i, j, k \in \{1, ..., p\}$, and $\omega_i > \omega^*$, $\forall i \in \{1, ..., p\}$, with ω^* large enough, ensures that the norm of the error vector e_y admits the following bound

$$\|e_y(t)\| \leq \beta(\|e_y(0)\|, t) + \alpha(\int_0^t \gamma(\|e_\Delta(s)\|)ds,$$

where $\alpha \in \mathcal{K}$, $\beta \in \mathcal{KL}$, $\gamma \in \mathcal{K}$, and $\|e_\Delta\|$ satisfies:

1-$(\frac{1}{\omega}, d)$-*Uniform Stability:* For every $c_2 \in]d, \infty[$, there exists $c_1 \in]0, \infty[$ and $\hat{\omega} > 0$ such that for all $t_0 \in \mathbb{R}$ and for all $x_0 \in \mathbb{R}^n$ with $||e_\Delta(0)|| < c_1$ and for all $\omega > \hat{\omega}$,

$$||e_\Delta(t, e_\Delta(0))|| < c_2, \ \forall t \in [t_0, \infty[$$

2-$(\frac{1}{\omega}, d)$-*Uniform ultimate boundedness:* For every $c_1 \in]0, \infty[$ there exists $c_2 \in]d, \infty[$ and $\hat{\omega} > 0$ such that for all $t_0 \in \mathbb{R}$ and for all $x_0 \in \mathbb{R}^n$ with $||e_\Delta(0)|| < c_1$ and for all $\omega > \hat{\omega}$,

$$||e_\Delta(t, e_\Delta(0))|| < c_2, \ \forall t \in [t_0, \infty[$$

3-$(\frac{1}{\omega}, d)$-*Global uniform attractively:* For all $c_1, \ c_2 \in (d, \infty)$ there exists $T \in]0, \infty[$ and $\hat{\omega} > 0$ such that for all $t_0 \in \mathbb{R}$ and for all $x_0 \in \mathbb{R}^n$ with $||e_\Delta(0)|| < c_1$ and for all $\omega > \hat{\omega}$,

$$||e_\Delta(t, e_\Delta(0))|| < c_2, \ \forall t \in [t_0 + T, \infty[$$

where d is given by: $d = min\{r \in]0, \infty[: \ \Gamma_H \subset B(\Delta, r)\}$, with $\Gamma_H = \{\hat{\Delta} \in \mathbb{R}^n : \ \|\frac{\partial Q(\hat{\Delta}, t)}{\partial \hat{\Delta}}\| < \sqrt{\frac{2\rho_Q}{ka\beta_0}}\}, \ 0 < \beta_0 \leq 1$, and $B(\Delta, r) = \{\hat{\Delta} \in \mathbb{R}^n : \ ||\hat{\Delta} - \Delta|| < r\}$.

Remark 7. Lemma 6 shows that the estimation error is bounded by a constant c_2 which can be tightened by making the constant d small. The d constant can be tuned by tuning the cardinal of the set Γ_H, which in turns can be made small by choosing large values for the coefficients a and k of the ES algorithm (10).

Proof.
Consider the system (6), (7), then under Assumption 1, the controller u_{iss} ensures that the tracking error dynamic (8) is iISS between the input e_Δ and the state vector e_y, which by Definition 1, implies that there exist functions $\alpha \in \mathcal{K}$, $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$, such that, for all $e(0) \in \mathcal{D}_e$ and $e_\Delta \in \mathcal{D}_{e_\Delta}$, the norm of the error vector e_Δ admits the following bound

$$\|e_y(t)\| \leq \beta(\|e_y(0)\|, t) + \alpha(\int_0^t \gamma(\|e_\Delta\|))ds \qquad (11)$$

for all $t \geq 0$.
Now, we need to evaluate the bound on the estimation vector $\tilde{\Delta}$, to do so we use the results presented in Scheinker [2013]. Indeed, based on Theorem 3 of Scheinker [2013], we can conclude under Assumption 4, that the estimator (10), makes the local optimum of Q; $\Delta^* = \Delta$ (see Assumption 3), $(\frac{1}{\omega}, d)$-SPUUB, where $d = min\{r \in]0, \infty[: \Gamma_H \subset B(\Delta, r)\}$, with $\Gamma_H = \{\hat{\Delta} \in \mathbb{R}^n : \ |\frac{\partial Q(\hat{\Delta}, t)}{\partial \hat{\Delta}}| < \sqrt{\frac{2\rho_Q}{ka\beta_0}}\}, \ 0 < \beta_0 \leq 1$, and $B(\Delta, r) = \{\hat{\Delta} \in \mathbb{R}^n : ||\hat{\Delta} - \Delta|| < r\}$, which by Definition 2 implies that $\|e_\Delta\|$ satisfies the three conditions: $(\frac{1}{\omega}, d)$-Uniform Stability, $(\frac{1}{\omega}, d)$-Uniform ultimate boundedness, and $(\frac{1}{\omega}, d)$-Global uniform attractively. □

Remark 8. The upper-bounds of the estimated parameters used in Lemma 6 are correlated to the choice of the extremum seeking algorithm (9) and (10), however, these bounds can be easily changed by using other ES algorithms, e.g. Noase et al. [2011], which is due to the modular design of the controller, that uses the iISS robust part to ensure boundedness of the error dynamics and the learning part to improve the tracking performance.

Remark 9. We point out here that ISS can be substituted for iISS if we are dealing with time-invariant systems and solving a regulation problem instead of a time-varying trajectory tracking.

4. CASE STUDY

We study here the example of electromagnetic actuators modelled with the nonlinear equations Peterson and Stefanopoulou [2004]

$$\begin{aligned} m\frac{d^2x}{dt^2} &= k(x_0 - x) - \eta\frac{dx}{dt} - \frac{ai^2}{2(b+x)^2} \\ u &= Ri + \frac{a}{b+x}\frac{di}{dt} - \frac{ai}{(b+x)^2}\frac{dx}{dt}, \ 0 \leq x \leq x_f \end{aligned} \qquad (12)$$

where, x represents the armature position physically constrained between the initial position of the armature 0, and the maximal position of the armature x_f, $\frac{dx}{dt}$ represents the armature velocity, m is the armature mass, k the spring constant, x_0 the initial spring length, η the damping coefficient, $\frac{ai^2}{2(b+x)^2}$ represents the electromagnetic force (EMF) generated by the coil, a, b being constant parameters of the coil, R the resistance of the coil, $L = \frac{a}{b+x}$ the coil inductance (assumed to be dependent on the position of the armature), $\frac{ai}{(b+x)^2}\frac{dx}{dt}$ represents the back EMF. Finally, i denotes the coil current, $\frac{di}{dt}$ its time derivative and u represents the control voltage applied to the coil. We address the control problem of the electromagnetic system with the following parameters uncertainties: the spring constant k , and the damping coefficient η.
Consider now the dynamical system (12), and let us define the state vector $\mathbf{z} := [z_1 \ \ z_2 \ \ z_3]^T = [x \ \ \dot{x} \ \ i]^T$. The objective of the control is to make the variables (z_1, z_2) robustly track a sufficiently smooth (at least C^2) time-varying position and velocity trajectories $z_1^{ref}(t)$, $z_2^{ref}(t) = \frac{dz_1^{ref}(t)}{dt}$ that satisfy the following constraints: $z_1^{ref}(t_0) = z_{1_{int}}$, $z_1^{ref}(t_f) = z_{1_f}$, $\dot{z}_1^{ref}(t_0) = \dot{z}_1^{ref}(t_f) = 0$, $\ddot{z}_1^{ref}(t_0) = \ddot{z}_1^{ref}(t_f) = 0$, where t_0 is the starting time of the trajectory, t_f is the final time, $z_{1_{int}}$ is the initial position and z_{1_f} is the final position.

To start, let us first write the system (12) in the following form

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= \frac{k}{m}(x_0 - z_1) - \frac{\eta}{m}z_2 - \frac{a}{2m(b+z_1)^2}z_3^2 \\ \dot{z}_3 &= -\frac{R}{\frac{a}{b+z_1}}z_3 + \frac{z_3}{b+z_1}z_2 + \frac{u}{\frac{a}{b+z_1}} \end{aligned} \qquad (13)$$

It was proven in Atinc and Benosman [2013], that the following controller

$$
\begin{aligned}
u = &\frac{a}{b+z_1}\Bigg(\frac{R(b+z_1)}{a}z_3 - \frac{z_2 z_3}{(b+z_1)}\\
&+\frac{1}{2z_3}\Big(\frac{a}{2m(b+z_1)^2}(z_2 - z_2^{ref}) - c_2(z_3^2 - \tilde{u})\Big)\Bigg)\\
&+\frac{2mz_2}{z_3}\Bigg(\frac{\hat{k}}{m}(x_0 - z_1) - \frac{\hat{\eta}}{m}z_2 + c_3(z_1 - z_1^{ref}) + c_1(z_2 - z_2^{ref})\\
&+\kappa_1(z_2 - z_2^{ref})\|\psi\|_2^2 - \dot{z}_2^{ref}\Bigg)\\
&+\frac{m(b+z_1)}{z_3}\Bigg(\Bigg(\frac{\hat{k}}{m}(x_0 - z_1) - \frac{\hat{\eta}}{m}z_2 - \frac{a}{2m(b+z_1)^2}z_3^2 - \dot{z}_2^{ref}\Bigg)\\
&\Bigg(c_1 + \kappa_1\|\psi\|_2^2 - \frac{\hat{\eta}}{m}\Bigg) - \frac{\hat{\eta}}{m}\dot{z}_2^{ref}\Bigg)\\
&+\frac{m(b+z_1)}{z_3}\Bigg(2\kappa_1(z_2 - z_2^{ref})\Bigg(\frac{(x_0 - z_1)(-z_2)}{m^2} +\\
&\frac{z_2\Big(\frac{\hat{k}}{m}(x_0 - z_1) - \frac{\hat{\eta}}{m}z_2 - \frac{az_3^2}{2m(b+z_1)^2}\Big)}{m^2}\Bigg)\Bigg)\\
&-\kappa_2(z_3^2 - \tilde{u})\Big|\frac{2m(b+z_1)^2}{a}\Big|^2\Bigg[\Big|c_1\\
&+\kappa_1\|\psi\|_2^2 - \frac{\hat{\eta}}{m}\Big|^2 + \Big|2\kappa_1(z_2 - z_2^{ref})\Big|^2\Big|\frac{z_2}{m^2}\Big|^2\Bigg]\|\psi\|_2^2\\
&-\kappa_3(z_3^2 - \tilde{u})\Big|\frac{2m(b+z_1)^2}{a}\Big|^2\|\psi\|_2^2\\
&+\frac{m(b+z_1)}{z_3}\Bigg(-\frac{\hat{k}}{m}z_2 - \ddot{z}_2^{ref} + c_3(z_2 - z_2^{ref})\Bigg)
\end{aligned}
\tag{14}
$$

with

$$
\begin{aligned}
\tilde{u} = &\frac{2m(b+z_1)^2}{a}\Bigg(\frac{\hat{k}}{m}(x_0 - z_1) - \frac{\hat{\eta}}{m}z_2 + c_3(z_1 - z_1^{ref})\\
&+c_1(z_2 - z_2^{ref}) - \dot{z}_2^{ref}\Bigg)\\
&+\frac{2m(b+z_1)^2}{a}\Bigg(\kappa_1(z_2 - z_2^{ref})\|\psi\|_2^2\Bigg)
\end{aligned}
\tag{15}
$$

where, $\hat{k},\ \hat{\eta}$ are the estimated values of the uncertain parameters, makes the system (13) iISS with respect to the estimation error input, which together with Lemma 6, ensures that the controller (14), and (15), with the ES algorithm

$$
\begin{aligned}
&Q(x_k, x_\eta) = q_1(z_1(t_f) - z_1(t_f)^{ref})^2 + q_2(z_2(t_f) - z_2^{ref}(t_f))^2\\
&\dot{x}_k = a\sqrt{(\omega_k)}cos(\omega_k t) - k\sqrt{\omega_k}sin(\omega_k t)Q(\hat{\Delta})\\
&\dot{x}_\eta = a\sqrt{(\omega_\eta)}cos(\omega_\eta t) - k\sqrt{\omega_\eta}sin(\omega_\eta t)Q(\hat{\Delta})\\
&\hat{k}(t) = k_{nominal} + x_k(t)\\
&\hat{\eta}(t) = \eta_{nominal} + x_\eta(t)
\end{aligned}
\tag{16}
$$

where, $k_{nominal},\ \eta_{nominal}$ are the nominal values of the parameters, with $\omega_k \neq \omega_\eta,\ k > 0,\ a > 0,\ q_1, q_2 > 0$, leads to bounded tracking and estimation errors.

We now illustrate numerically our approach using the system parameters given in Table 1 Kahveci and Kolmanovsky [2010]. The reference trajectory is designed to be a 5^{th} order polynomial, $x^{ref}(t) = \sum_{i=0}^{5} a_i(\frac{t}{t_f})^i$ where the coefficients a_i are selected such that the following conditions are satisfied: $x^{ref}(0) = 0.2\ mm,\ x^{ref}(0.5) = 0.7\ mm,\ \dot{x}^{ref}(0) = 0,\ \dot{x}^{ref}(0.5) = 0, \ddot{x}^{ref}(0) = 0,\ \dot{x}^{ref}(0.5) = 0$. We consider the uncertainties given by $\Delta k = -5\ \frac{N}{mm}$, and $\Delta\eta = -1\ \frac{kg}{s}$. To make the simulation case more challenging we also introduced an initial error $x(0) = 0.01\ mm$ on the armature position. We implemented the controller (14) and (15) with the coefficients $c_1 = 100,\ c_2 = 100,\ c_3 = 2500,\ \kappa_1 = \kappa_2 = \kappa_3 = 0.25$, together with the learning algorithm (16) with the coefficients $k = 0.1,\ a = 0.1,\ \omega_\eta = 0.4\ rad/sec,\ \omega_k = 0.5\ rad/sec$. Here due to the cyclic nature of the problem, i.e., cyclic motion of the armature between 0 and x_f, the uncertain parameters estimate vector $(\hat{k},\ \hat{\eta})^T$ is updated for each cycle, i.e., at the end of each cycle at $t = t_f$, the cost function Q is updated, and the new estimate of the parameters is computed for the next cycle. The purpose of using MES scheme along with iISS-backstepping controller is to improve the performance of the iISS-backstepping controller by better estimating the system parameters over many cycles, hence decreasing the error in the parameters over time to provide better trajectory following for the actuator. As can be seen in Figures 1 and 2, the robustification of the backstepping control via extremum seeking greatly improves the tracking performance. Figures 3 and 4 show that the cost function starts at 15, reaches to half of the initial value within 20 iterations, and decreases rapidly afterwards. Moreover, the estimated parametric uncertainties $\Delta k,\ \Delta\eta$ converge to regions around the actual parameter values, as shown on Figures 5 and 6. The number of iterations for the estimate to reach the actual value of the parameters may appear to be high. The reason behind that is that the allowed uncertainties in the parameters are large, hence the extremum seeking scheme requires a lot of iterations to improve performance. Furthermore, we purposely tested the challenging case of multiple simultaneous uncertainties, which makes the space search for the learning algorithm large (note that this case of multiple uncertainties could not be solved with other classical model-based adaptive controller Benosman and Atinc [2013b], due to some intrinsic limitations of model-based adaptive controller). However, in real-life applications uncertainties accumulate gradually over a long period of time, while the learning algorithm keeps tracking these changes continuously. Thus, the extremum seeking algorithm will be able to improve the controller performance quickly, meaning that it will enhance the backstepping control within fewer iterations. Finally, the control voltage is depicted on Figure 7, which shows an initial high value due to the relatively large simulated initial condition error on the armature position.

Parameter	Value
m	0.27 [kg]
R	6 [Ω]
η	7.53 [$\frac{kg}{s}$]
x_0	8 [mm]
k	158 [$\frac{N}{mm}$]
a	$14.96 \times 16^{\ \ 6}$ [$\frac{Nm^2}{A^2}$]
b	$4 \times 10^{\ \ 5}$ [m]

Table 1. System Parameter Values

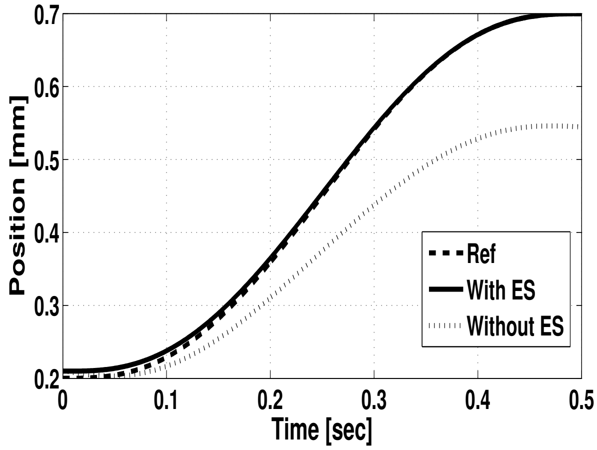

Fig. 1. Obtained Armature Position vs. Reference

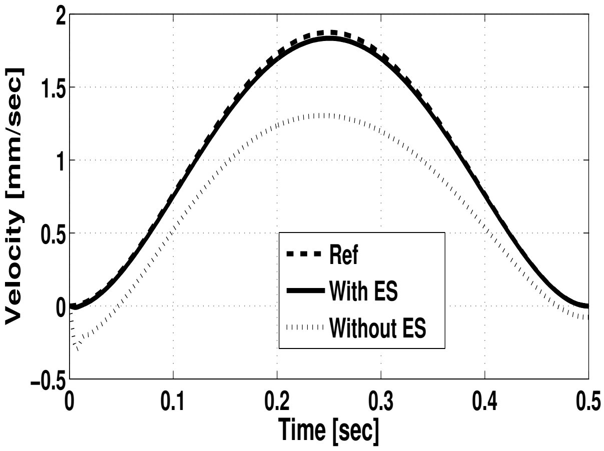

Fig. 2. Obtained Armature Velocity vs. Reference Trajectory

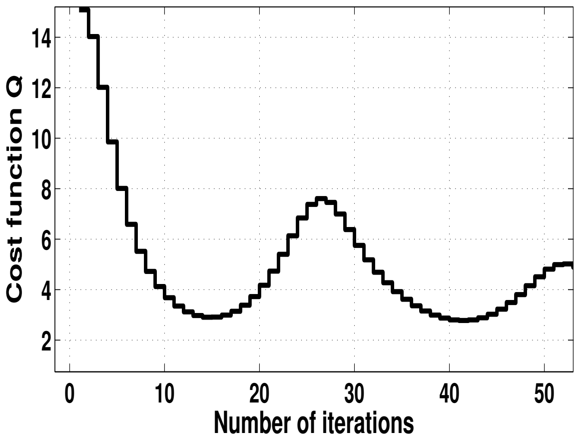

Fig. 3. Cost function- zoom

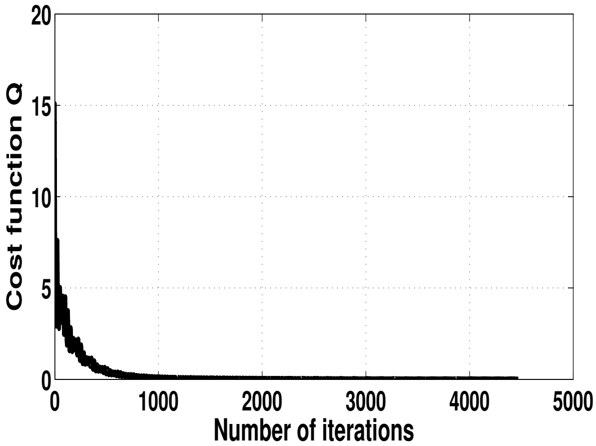

Fig. 4. Cost function

5. CONCLUSION

In this paper we have studied the problem of ES-based indirect adaptive control for nonlinear systems with parametric uncertainties. We argued that for the class of nonlinear systems which can be rendered iISS w.r.t. the parameter estimation errors by a robust feedback controller, it is possible to combine the iISS feedback controller with a model-free ES algorithm to obtain a learning-based adaptive controller. We showed an application of this approach on a mechatronic example and reported encouraging numerical results. In this preliminary paper, we introduced the idea in a general setting, however, further investigation are needed to analyze specific nonlinear systems classes which can be stabilized (in the iISS sense) w.r.t. to the

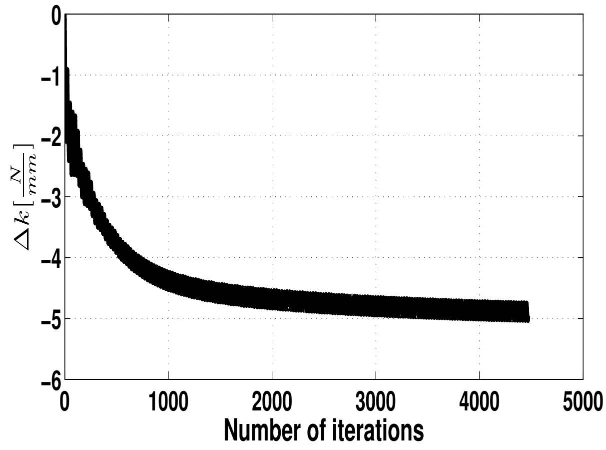

Fig. 5. Parameter k estimate

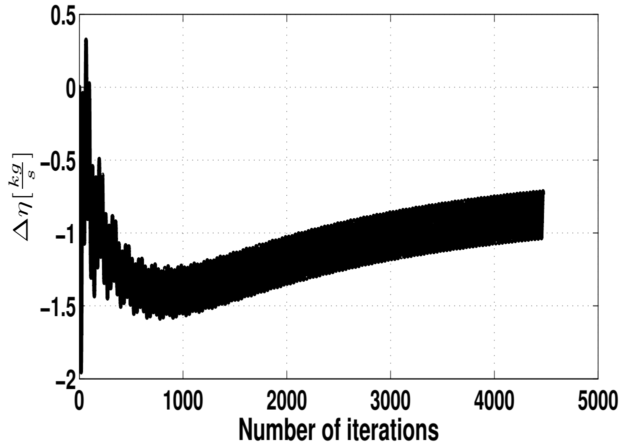

Fig. 6. Parameter η estimate

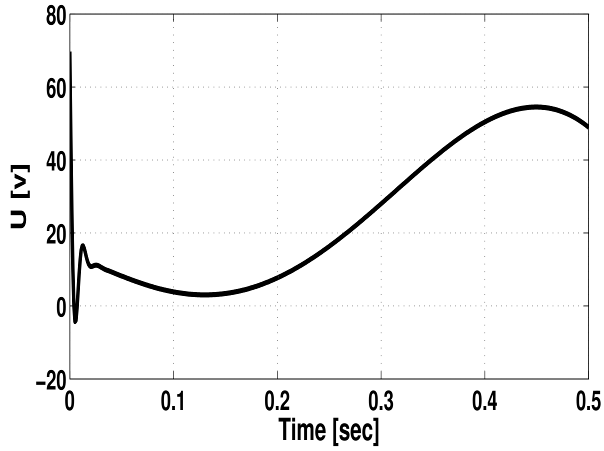

Fig. 7. Control voltage- Case with uncertain k, η, f_d

estimation error of the uncertain parameters, and show for these specific classes a constructive control design approach, in the context of the learning-based adaptive control presented here. Further work will also deal with using different ES algorithms with less restrictive conditions on the dither signals amplitude and frequencies, e.g. Guay et al. [2013], and comparing the obtained controllers to the available classical adaptive controllers.

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