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Xinying Zhang, Andreas F. Molisach, Sun-Yuan Kung

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Xinying Zhang
Princeton University
Princeton, NJ 08544
xinying@ee.princeton.edu

Andreas F. Molisch
Senior Member, IEEE
Mitsubishi Electric Research Lab
Murray Hill, NJ
Andreas.Molisch@ieee.org

Sun-Yuan Kung
Fellow, IEEE
Princeton University
Princeton, NJ 08544
kung@ee.princeton.edu

***Abstract*— This paper addresses the antenna subset selection in multiple antenna systems with full diversity transmission, for both correlated and uncorrelated channels. To reduce the severe performance degradation of traditional selection/combining schemes, we propose to embed phase-shift-only operations in the RF chains before selection. The resulting system shows a significant advantage in utilizing the multiple antenna diversity under almost any channel condition while incurring only a small hardware overhead. With the optimum phase shifter design given in analytical form, our analysis shows that with more than two branches allowed for selection, the new scheme can achieve the same SNR gain as the full-complexity MRC (Maximum-Ratio-Combining). Even when only one branch is allowed, the performance is still well above the conventional selection scheme and near optimum.**

I. Introduction

MIMO (Multiple-Input-Multiple-Output) systems have attracted considerable attention, due to the promising performance enhancement via deploying multiple antennas at both the transmission ends. While a lot of attention has been given to the rate increase through simultaneous transmission of different data streams [11][12], there is also great interest in using the multiple antenna elements for performance enhancement of a single data stream, by providing higher diversity.

The high expense of implementing the multiple RF chains motivates the recent popularity of antenna selection schemes [2][9], which optimally chooses a subset from the transmit and/or receive antennas for transmission and therefore maximally benefits from the multiple antenna diversities within the RF cost constraint. Despite the great advantages in terms of cost reduction, they suffer from severe performance degradation. In most practical MIMO channels, due to the directional nature of the multipath propagation, the signals at the antenna array are highly correlated dependent on the arriving angles. In this case, the performance of conventional selection reduces to that of a L-antenna system, losing all advantages of having additional antenna elements. Even in uncorrelated channels, the performance degradation (due to smaller average SNR) can be significant when only a small portion of the antennas are allowed for selection. In a recent paper [3], we have proposed a scheme that addresses the first problem, by inserting a Butler matrix (a hardwired spatial FFT operation in the RF domain) between the antenna elements and the selection switch.

In this paper, we extend and modify our results to deal with both correlated and independent channel fading by embedding a variable phase-shift-only operation that performs an optimum transformation before selection. In contrast to the FFT operation proposed in [3], the variable phase shifters used here is adapting to the channel. We recognize the rapid advances in MMIC techniques, and stress that the economic design and fabricate of large-scale circuity with variable phase shifters are available for the microwave frequency range [5][6]. Our analysis shows that under diversity transmission, with more than two branches allowed for selection, the new scheme can achieve the same performance as the full-complexity reception with all the signals involved. When only one branch is allowed, the SNR gain of the new scheme is also significantly better than the conventional one and near optimum. For simplicity we only address the receiver selection while the transmit selection can be treated in duality.

II. Spatially Correlated Channel Model

We consider a general multiple antenna system adopting ULA (Uniformly-spaced Linear Antenna) arrays at both the transmitter and the receiver sides. We denote t and r as the number of transmit and receive antenna elements respectively, and $\mathbf{H}$ is the $r \times t$ transfer function of the MIMO channel. The channel fading is assumed to be block-fading, which remains fixed over a block of symbols and then changes to a new, independent realization. Considering the spatial correlation among different antennas, we adopt the channel model that has been extensively used in [4] [8] [9]:

$$\mathbf{H} = \mathbf{R}^{1/2}\mathbf{W}\mathbf{T}^{1/2}, \tag{1}$$

where $\mathbf{W}$ is a Rayleigh fading matrix with i.i.d. circularly symmetric complex Gaussian entries $\sim \mathcal{N}_C(0,1)$, $\mathbf{R}$, $\mathbf{T}$ are $r \times r$, $t \times t$ matrices denoting receive and transmit correlations respectively. The two correlation matrices are determined by the angles of arrival (AoA) and departure (AoD). For simplicity of analysis we only consider the receiver correlation and set $\mathbf{T} = \mathbf{I}_t$, and the PAS (Power Azimuth Spectrum) of the AoA is assumed to be Gaussian distributed: $\theta = \theta_r + \epsilon;\ \epsilon \sim \mathcal{N}(0, \sigma_r^2)$. The assumptions above allow a closed-form computation of

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matrix $\mathbf{R}$, as shown in [13]:

$$\mathbf{R}_{m,n} = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_r} e^{-j2\pi(n-m)d\cos(\theta_r+\epsilon)} e^{-\frac{\epsilon^2}{2\sigma_r^2}} d\epsilon, \quad (2)$$

where d is the relative (receive) antenna spacing with respect to the carrier wavelength.

III. Antenna Selection Systems and Comparison

The block diagram of our considered MIMO diversity system is illustrated in Figure 1. The information stream is multiplied by a t-dimensional complex weighting vector $\vec{v}$, converted to the passband and applied to each of the t transmitting antennas. At the receiver end, the observation streams are demodulated and then linearly combined with the weighting vector $\vec{u}$ to get an estimate of the information stream. Without antenna selection, all the r receiving streams are combined in the full-complexity reception, which requires r down converters. In conventional antenna selection, L out of the r streams are chosen for combination. In our new scheme, the observations are first passed through a $r \times r$ matrix $\mathbf{\Phi}$ with phase-shift-only entries before selection and combination.

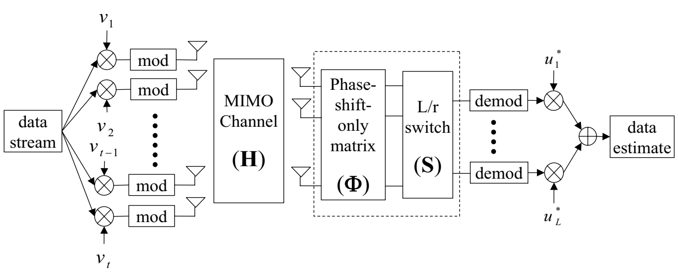


Fig. 1. MIMO channel model and system diagram.

The system described above is expressed by the linear equation:

$$\vec{x}(k) = \mathbf{H}\vec{v}s(k) + \vec{n}(k), \quad (3)$$

where $s(k) \in \mathcal{C}$ is the transmitting stream, $\vec{x}(k) \in \mathcal{C}^r$ is the sample stacks of the complex-valued receiver data sequences. The total transmission power is constrained to P: $\mathcal{E}[s(k)s^*(k)] = P$. The thermal noises $\vec{n}(k) \in \mathcal{C}^r$ are i.i.d. circularly symmetric Gaussian random processes with variance $\sigma_n \mathbf{I}_r$. And the t-dimensional transmitter weighting vector satisfies $\|\vec{v}\| = 1$.

A. Full-Complexity MRC (FC-MRC) Scheme

When all the antenna elements are exploited, the output signal at the receive linear combiner $\vec{u}$ is

$$\hat{s}(k) = \vec{u}^*\mathbf{H}\vec{v}s(k) + \vec{u}^*\vec{n}(k), \quad (4)$$

where $*$ denotes the conjugate transpose of a matrix (vector). The estimated SNR (Signal-to-Noise Ratio) after linear combining is therefore

$$\frac{\mathcal{E}[|\vec{u}^*\mathbf{H}\vec{v}s(k)|]^2}{|\vec{u}^*\vec{n}(k)|^2} = \rho\frac{|\vec{u}^*\mathbf{H}\vec{v}|^2}{|\vec{u}^*|^2}, \quad (5)$$

where $\rho = \frac{P}{\sigma_n}$ is the average SNR. To facilitate the performance analysis we introduce the singular value decomposition (SVD) of the transfer function $\mathbf{H}$: $\mathbf{H} = \mathbf{U_H \Sigma_H V_H^*}$, where $\mathbf{U_H}$ and $\mathbf{V_H}$ are unitary matrices representing the left and right singular vector spaces of $\mathbf{H_H}$, respectively; and $\mathbf{\Sigma_H}$ is the diagonal matrix consisting of all the singular values of $\mathbf{H}$. For convenience we denote $\lambda_{\mathbf{H,i}}$ as the i-th largest singular value of matrix $\mathbf{H}$, and $\vec{u}_{\mathbf{H,i}}, \vec{v}_{\mathbf{H,i}}$ are the left and right singular vectors of $\mathbf{H}$ associated with $\lambda_{\mathbf{H,i}}$, respectively.

To maximize the SNR upon reception, MRT (Maximum-Ratio-Transmission) and MRC (Maximum-Ratio-Combining) should be adopted with the optimum weights $\vec{u} = \vec{u}_{\mathbf{H,1}}$, $\vec{v} = \vec{v}_{\mathbf{H,1}}$, which result in an optimum achievable SNR

$$SNR_{FC} = \rho\lambda_{\mathbf{H,1}}^2. \quad (6)$$

B. Hybrid-Selection MRC (HS-MRC) Scheme

When only L out of the r ($L < r$) receive antennas are selected and combined, mathematically, each selection option corresponds to a $L \times r$ selection matrix on the transfer function, which extracts L out of the r rows in $\mathbf{H}$ that are associated with the selected antennas. We denote $\mathcal{S}_L$ as the set of all such selection matrices. For any selection option $\mathbf{S} \in \mathcal{S}_L$, an optimum estimate SNR is achieved via a similar MRC on the L branches:

$$\max_{\vec{u}^* \in \mathcal{C}^{1\times L}} \max_{\|\vec{v}\|=1} \rho\frac{\|\vec{u}\mathbf{S}\mathbf{H}\vec{v}\|^2}{\|\vec{u}\|^2} = \rho\lambda_{\mathbf{SH},1}^2. \quad (7)$$

The L/r HS-MRC reception chooses the optimal antenna subset selection matrix $\mathbf{S}$ among all the elements in $\mathcal{S}_L$ that maximizes the estimate SNR above:

$$SNR_{HS}(L) = \max_{\mathbf{S}\in\mathcal{S}_L} \rho\lambda_{\mathbf{SH},1}^2. \quad (8)$$

C. FFT-based Selection MRC (FFTS-MRC) Scheme

To cope with the highly correlated MIMO channels, a FFT-based selection scheme is proposed in [3], where a r-point FFT matrix $\mathbf{\Phi}_{FFT}$ is inserted in the RF chain before the L/r receiver antenna selection $\mathbf{S}$ ($\mathbf{\Phi}_{FFT}$ is normalized: $\mathbf{\Phi}_{FFT}\mathbf{\Phi}_{FFT}^* = \mathbf{I}_r$ to preserve the noise level). The FFT in microwave frequency can be implemented by a hardwired Butler matrix and the L/r selection is followed by a MRC on the L selected branches after FFT. The optimum estimate SNR achieved in FFTS is:

$$SNR_{FFTS}(L) = \max_{\mathbf{S}\in\mathcal{S}_{\mathcal{L}}} \rho\lambda_{\mathbf{S\Phi}_{FFT}\mathbf{H},1}^2. \quad (9)$$

D. Phase-Shift & Selection MRC (PSS-MRC) Scheme

The phase-shift matrix $\mathbf{\Phi}$ and the L/r selection $\mathbf{S}$ in Figure 1 can be integrated into one $L \times r$ matrix with phase-shift-only entries. The set consisting of all such matrices is denoted by $\mathcal{F}_L = \{\mathbf{F}|\ [\mathbf{F}]_{m,n} = e^{j\phi_{m,n}},\ 1 \le m \le L;\ 1 \le n \le r\}$. In PSS, given any $\mathbf{F} \in \mathcal{F}_L$, the linear combining is performed on the virtual channel $\mathbf{FH}$ with the thermal noises $\mathbf{F}\vec{n}(k)$. Following a similar argument as before, with the optimal choice of $\mathbf{F} \in \mathcal{F}_L$, the maximal SNR achieved in PSS scheme is

$$SNR_{PSS}(L) = \max_{\mathbf{F}\in\mathcal{F}_L} \max_{\vec{u}^*\in\mathcal{C}^{1\times L}} \max_{\|\vec{v}\|=1} \rho \frac{|\vec{u}^*\mathbf{F}\mathbf{H}\vec{v}|^2}{\|\vec{u}^*\mathbf{F}\|^2}. \qquad (10)$$

The PSS also consists of an optimal selection search as in HS-MRC, in the space of $\mathcal{F_L}$. For any fixed $\mathbf{F} \in \mathcal{F_L}$, however, the optimal weighting vector and estimate SNR can no longer be treated via a straightforward svd operation, as the matrix $\mathbf{F}$ is in general non-unitary and does not preserve the whiteness of noises. Instead, the SNR is computed based on a standard Gram-Schmidt procedure. Assume the rank of $\mathbf{F}$ is k ($k \le L$), then construct a $k \times r$ matrix $\mathbf{Q}$ whose k row vectors form a set of orthonormal basis for the row subspace of $\mathbf{F}$. The estimate SNR with optimal choice is then

$$\begin{aligned} & \max_{\vec{u}^*\in\mathcal{C}^{1\times L}} \max_{\|\vec{v}\|=1} \rho \frac{|\vec{u}^*\mathbf{F}\mathbf{H}\vec{v}|^2}{\|\vec{u}^*\mathbf{F}\|^2} \\ = & \max_{\vec{u}^*\in\mathcal{C}^{1\times k}} \max_{\|\vec{v}\|=1} \rho \frac{|\vec{u}^*\mathbf{Q}\mathbf{H}\vec{v}|^2}{\|\vec{u}^*\mathbf{Q}\|^2} = \rho\lambda_1^2(\mathbf{QH}). \qquad (11) \end{aligned}$$

E. Unification and Performance Comparisons via Subspace Explanation

The aforementioned four schemes can be unified from a subspace perspective. The optimum SNRs in (6), (8), (9) and (10) can be equivalently written in the same formula:

$$\max_{\vec{u}^*\in\mathcal{U}} \max_{\|\vec{v}\|=1} \rho \frac{|\vec{u}^*\mathbf{H}\vec{v}|^2}{\|\vec{u}\|^2}, \qquad (12)$$

with different allowable set $\mathcal{U}$. Based on the previous derivations, it is straightforward to show that the constrained space for the four schemes are

1) FC-MRC: $\mathcal{U_{FC}} = \mathcal{C}^{1\times r}$;
2) HS-MRC: $\mathcal{U_{HS}} = \cup_{\mathbf{S}\in\mathcal{S}_L} R.S.(\mathbf{S})$ where $R.S.(\mathbf{S})$ is the row span of $\mathbf{S}$;
3) FFTS-MRC: $\mathcal{U_{FFTS}} = \cup_{\mathbf{S}\in\mathcal{S}_L} R.S.(\mathbf{S}\mathbf{\Phi}_{FFT})$;
4) PSS-MRC: $\mathcal{U_{PSS}} = \cup_{\mathbf{F}\in\mathcal{F}_L} R.S.(\mathbf{F})$.

The four selection/combining schemes attempt to maximize the same function in (12) with $\vec{u}^*$ chosen from the corresponding candidate spaces related by $\mathcal{U_{HS}}, \mathcal{U_{FFTS}} \subset \mathcal{U_{PSS}} \subset \mathcal{U_{FC}}$, resulting in an estimate SNR increasing in the following order:

$$SNR_{HS}, SNR_{FFTS} \le SNR_{PSS} \le SNR_{FC}. \qquad (13)$$

Due to the marked space expansion from $\mathcal{U_{HS}}$ to $\mathcal{U_{FC}}$, FC-MRC performs much better than HS-MRC, regardless of the spatial correlation level in the channel: $SNR_{FC} \gg SNR_{HS}$. Recall that the optimum SNR in FC-MRC is achieved with the weighting vector $\vec{u} = \vec{u}_{\mathbf{H},\mathbf{1}}$. Therefore the equality $SNR_{PSS} = SNR_{FC}$ can be achieved if and only if $\vec{u}_{\mathbf{H},\mathbf{1}} \in \mathcal{U_{PSS}}$, i.e. $\vec{u}_{\mathbf{H},\mathbf{1}}$ is a vector with phase-shift-only entries or a linear combination of L such vectors. As will be shown below, when $L \ge 2$, this condition is always guaranteed, which implies that

$$SNR_{PSS} = SNR_{FC} \text{ when } L \ge 2. \qquad (14)$$

When $L = 1$, $\mathcal{U_{PSS}}$ offers a sufficiently large space to approximate $\vec{u}_{\mathbf{H},\mathbf{1}}$, leading to a significant improvement over HS-MRC.

F. Hardware Comparison

The hardware expense of the antenna selection/combining schemes is dominated by the three factors: (1) RF to baseband demodulators, (2) RF chain operations, and (3) the optimum selection matrix search.

In contrast to the expenses of r downconverters in FC-MRC, only L such devices are required for HS-MRC, FTTS-MRC and PSS-MRC. The saving is prominent especially when $L \ll r$. As for RF components, the FFTS requires a Butler matrix, which can be easily implemented in microwave frequency, e.g., by delay lines. Our novel PSS scheme benefits from recent advances in controllable RF components that make feasible the implementation of fast phase-shift-only operations in RF chains with a minor hardware overhead [5] [6].

For optimum subset selection, HS-MRC and FFTS-MRC search in the space $\mathcal{S}_L$, demanding $|\mathcal{S}_L| = \binom{r}{L}$ svd operations. A faster (but suboptimum) selection algorithm is now available for HS-MRC [7]. For PSS, as will be addressed in the next section, the optimum (or suboptimum) selection matrix $\mathbf{F}$ is given in closed-form when the channel state information (CSI) is tracked at the receiver. The calculation in PSS hence requires no search in the full space $\mathcal{F}_L$. Only a QR factorization (or, equivalently, a Gram-Schmidt procedure) is in demand together with a svd operation. Due to the advances in fast digital signal processing, the computational load of PSS-MRC operations is not a significant obstacle.

IV. Optimum Phase Shifter Design of PSS Reception

In this section we investigate the optimum design of the phase-shift selection matrix $\mathbf{F} \in \mathcal{F}_L$ and SNR performance in PSS scheme when the CSI (Channel State Information) is available at the receiver.

A. Pure LoS (Line-of-Sight)

We start from the extreme case $\sigma_r = 0$. This corresponds to the ideal LoS situation in which the PAS concentrates on one explicit spatial direction with no angle spread. The correlation matrix then collapses to a rank-1 matrix $\mathbf{R} = \vec{a}(\theta_r)\vec{a}^*(\theta_r)$, where $\vec{a}(\theta_r)$ is the antenna response vector defined as

$$\vec{a}(\theta_r) = \begin{bmatrix} 1 & e^{j2\pi d\cos\theta_r} & \ldots & e^{j2\pi(r-1)d\cos\theta_r} \end{bmatrix}^T. \quad (15)$$

The channel transfer function is therefore simplified to

$$\mathbf{H} = \mathbf{R}^{1/2}\mathbf{W} = \vec{a}(\theta_r)\vec{w}^*, \qquad (16)$$

where $\vec{w}$ is a $t \times 1$ random vector with i.i.d. complex Gaussian entries: $\vec{w} \sim \mathcal{N_C}(0, \mathbf{I}_t)$.

Without antenna selection, for any channel realization with $\vec{w}$, the FC-MRC just performs the beam forming in the direction of θ_r with the transmit/receive weighting vectors $\vec{v} = \frac{\vec{w}}{\|\vec{w}\|}$ and $\vec{u} = \vec{a}(\theta_r)$. The optimum estimate SNR of FC-MRC is

$$\begin{aligned} SNR_{FC} &= \rho \frac{|\vec{u}^*\vec{a}(\theta_r)\vec{w}^*\vec{v}|^2}{\|\vec{u}\|^2} \\ &= \rho r\|\vec{w}\|^2 \sim \Gamma(t, \rho r). \qquad (17) \end{aligned}$$

Here $\Gamma(t, \rho r)$ denotes the Gamma distribution with parameters t and ρr [1]. The distribution of SNR_{FC} in the last equality is obtained due to the fact that the 2-norm of the i.i.d. Gaussian random vector $\vec{w}$ follows the Gamma distribution with expectation t and variance t.

In the ideal LoS case, the optimum weights $\vec{u} = \vec{a}(\theta_r)$ of FC-MRC consists of phase-shift-only entries, which implies $\vec{a}^*(\theta_r) \in \mathcal{U}_{PSS}(L)$ for any $1 \leq L \leq r$. According to the discussion in Section III-E, the same SNR gain of FC-MRC can also be achieved by PSS-MRC in this case:

$$SNR_{PSS}(L) = SNR_{FC} = \rho r \|\vec{w}\|^2 \sim \Gamma(t, \rho r). \tag{18}$$

Even with only one branch selected ($L = 1$), the PSS-MRC can achieve the full channel diversity gain of freedom r. A simple design of the optimum phase-shift selection matrix $\mathbf{F} \in \mathcal{F}_L$ is to set one row of $\mathbf{F}$ as $\vec{a}^*(\theta)$ with all the other rows arbitrarily selected.

To demonstrate the performance improvement of PSS-MRC over HS-MRC, we calculate the SNR gain of HS-MRC reception:

$$\begin{aligned} SNR_{HS}(L) &= \max_{\mathbf{S} \in \mathcal{S}_L} \rho \lambda^2_{\mathbf{S}\vec{a}(\theta)\vec{w}^*,1} \\ &= \max_{\mathbf{S} \in \mathcal{S}_L} \rho \|\vec{w}\|^2 \|\mathbf{S}\vec{a}(\theta)\|^2 \\ &= \rho L \|\vec{w}\|^2 \sim \Gamma(t, \rho L). \end{aligned} \tag{19}$$

HS-MRC can only obtain a diversity gain of L.

B. General Case Analysis

In Section III-E, we have established that the PSS-MRC can deliver the same SNR as FC-MRC reception if and only if $\vec{u}^*_{\mathbf{H},1} \in \mathcal{U}_{PSS}(L)$, which spans all the possible linear combinations of L phase-shift-only vectors. This condition is satisfied in the ideal LoS channels with $\sigma_r = 0$; but for most general MIMO channels, the optimum antenna selection/combining could not be treated via a simple beam forming and the entries of $\vec{u}_{\mathbf{H},1}$ no longer lie on a circle. However, the analysis below shows that in this case, the full SNR gain of FC-MRC can still be achieved by PSS-MRC provided $L \geq 2$. The optimum design of the phase-shifters is given as a function of $\vec{u}_{\mathbf{H},1}$. When $L = 1$, there is in general an inevitable SNR loss with PSS-MRC. A suboptimal phase-shifter design for $L = 1$ is given in analytical form; such a design is supported by the simulation results as a near-optimum solution.

1) $L \geq 2$:
Given any complex vector $\vec{u}_{\mathbf{H},\mathbf{1}}$ of dimension r, we can always find two vectors with phase-shift-only entries whose linear combination is equal to $\vec{u}_{\mathbf{H},\mathbf{1}}$. Denote the entries of the singular vector as $\vec{u}_{\mathbf{H},\mathbf{1}} = \begin{bmatrix} \gamma_1 e^{j\varphi_1} & \gamma_2 e^{j\varphi_2} & \dots & \gamma_r e^{j\varphi_r} \end{bmatrix}^T$ and the phases of $\mathbf{F}$ as $\mathbf{F}_{m,n} = e^{j\phi_{m,n}}$ $(1 \leq m \leq 2;\ 1 \leq n \leq r)$. It is straightforward to show that $\vec{u}^*_{\mathbf{H},\mathbf{1}}$ can be linearly combined by the two row vectors of $\mathbf{F}$ if the weights and phases are set as follows:

$$\begin{aligned} &\begin{bmatrix} \frac{\gamma_{max}+\gamma_{min}}{2} & \frac{\gamma_{max}-\gamma_{min}}{2} \end{bmatrix} \mathbf{F} = \vec{u}^*_{\mathbf{H},1}, \quad \text{with} \\ &\gamma_{max} = \max_{1 \leq k \leq r} \gamma_k;\ \gamma_{min} = \min_{1 \leq k \leq r} \gamma_k, \\ &\phi_{1,i} = -\varphi_i - \cos^{-1} \frac{\gamma_i^2 + \gamma_{max}\gamma_{min}}{\gamma_i(\gamma_{max} + \gamma_{min})}; \\ &\phi_{2,i} = -\varphi_i + \cos^{-1} \frac{\gamma_i^2 - \gamma_{max}\gamma_{min}}{\gamma_i(\gamma_{max} - \gamma_{min})}. \end{aligned} \tag{20}$$

The $\mathbf{F}$ selection above guarantees $\vec{u}^*_{\mathbf{H},1} \in R.S.(\mathbf{F})$. Two phase-shift-only vectors are sufficient to span any vector $\vec{u}_{\mathbf{H},1}$; when $L > 2$, $\mathbf{F}$ can be selected in the same manner with other $L-2$ rows arbitrarily designed. Therefore with $L \geq 2$, we always have $\vec{u}^*_{\mathbf{H},1} \in \mathcal{U}_{PSS}(L)$, which in turn leads to the full SNR gain achieved by FC-MRC:

$$SNR_{PSS}(L) = SNR_{FC}, \quad L \geq 2. \tag{21}$$

2) $L = 1$:
When $L = 1$, the matrix $\mathbf{F} \in \mathcal{F}_L$ reduces to a $1 \times r$ phase-shift vector, denoted by $\mathbf{F} = \begin{bmatrix} e^{j\phi_1} & e^{j\phi_2} & \dots & e^{j\phi_r} \end{bmatrix}$. Recall (10), the optimum estimate SNR is now

$$\begin{aligned} &\max_{\mathbf{F} \in \mathcal{F}_1} \max_{u \in \mathcal{C}} \max_{\|\vec{v}\|=1} \rho \frac{|u\mathbf{F}\mathbf{U_H}\mathbf{\Sigma_H}\mathbf{V}^*_{\mathbf{H}}\vec{v}|^2}{\|u\mathbf{F}\|^2} \\ &= \max_{\mathbf{F} \in \mathcal{F}_1} \frac{\rho}{r} \sum_{i=1}^{t} |\mathbf{F}\vec{u}_{\mathbf{H},\mathbf{i}}|^2 \lambda^2_{\mathbf{H},\mathbf{i}}. \end{aligned} \tag{22}$$

As $\lambda_{\mathbf{H},\mathbf{1}} \geq \lambda_{\mathbf{H},\mathbf{2}} \geq \dots \geq \lambda_{\mathbf{H},\mathbf{t}}$, the maximization of the equation above could empirically be approximated by the optimization of $|\mathbf{F}\vec{u}_{\mathbf{H},\mathbf{1}}|^2$:

$$\max_{\mathbf{F} \in \mathcal{F}_1} |\mathbf{F}\vec{u}_{\mathbf{H},\mathbf{1}}|^2 = \max_{\mathbf{F} \in \mathcal{F}_1} |\sum_{k=1}^{r} \gamma_k e^{j\phi_k} e^{j\varphi_k}|^2 = (\sum_{k=1}^{r} \gamma_k)^2,$$

with the choice $\mathbf{F} = \begin{bmatrix} e^{-j\varphi_1} & \dots & e^{-j\varphi_r} \end{bmatrix}$.

V. Simulation

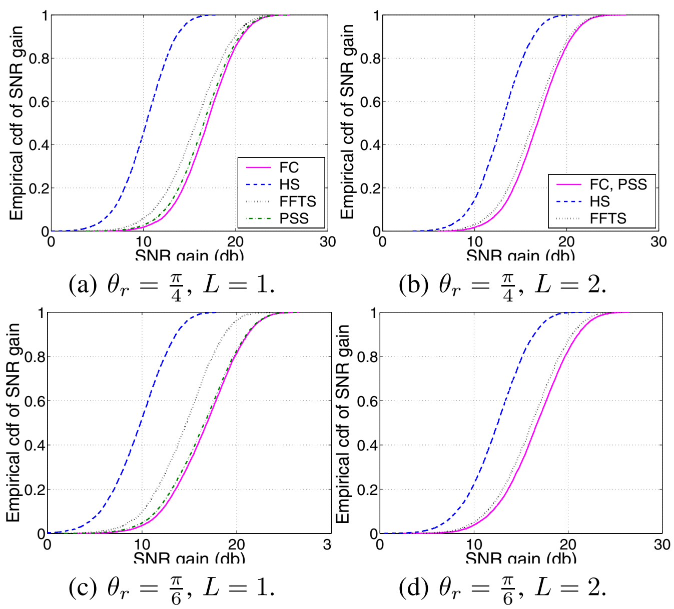
Simulations are conducted based on Monte Carlo tests. The SNR gain, defined as the ratio of the optimum estimate SNR to the average SNR $\rho = \frac{P}{N}$, serves as the major measurement to evaluate the performance of the different selection-combining schemes. The simulation is based on the channel model in Section II. As shown in [13], measurements have been done attempting to validate the Gaussian distribution of AoA; for different distance and environments the angle spread σ_r lies in the interval $[0, 6°]$. With small angle spread, the calculation of $\mathbf{R}$ in (2) is simplified to [13]:

$$\mathbf{R}_{m,n} \approx e^{-j2\pi(n-m)d\cos\theta_r} e^{-\frac{1}{2}[2\pi(n-m)d\sigma_r \sin\theta_r]^2}.$$

Both transmit and receive correlation are considered in the tests; the transmit correlation matrix is generated in a similar fashion. Figure 2 shows the empirical cdf of SNR gain for a MIMO system with $t = r = 8$ antenna elements at both ends and the antenna spacing is $d = 0.5$ carrier wavelength. Two results are plotted at $\theta_r = \frac{\pi}{4}$ and $\theta_r = \frac{\pi}{6}$. When $L = 1$, the

[1] The pdf. of $X \sim \Gamma(\alpha, \beta)$ is $p_X(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}$ $(x > 0)$. The Γ function is $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$. For a positive integer α, $\Gamma(\alpha) = (\alpha - 1)!$.

HS-MRC delivers a significant SNR loss of about $6 \sim 7\mathrm{db}$, as shown in (a) and (c). FFTS reception outperforms HS-MRC. The improvement is keenly dependent on the arriving angle: it is larger at $\theta_r = \frac{\pi}{4}$ in (a), where the FFTS is only $1 \sim 2\mathrm{db}$ from the optimum FC-MRC curve; at $\theta_r = \frac{\pi}{6}$, the loss of FFTS is more significant. At both angles, PSS behaves very close to the full-complexity curve. When $L = 2$, the SNR curves of PSS-MRC and FC-MRC overlap each other. In all the cases PSS-MRC proves to be the best among all antenna selection schemes. Figure 3 demonstrates the SNR gain of the four schemes with respect to the antenna spacing. We see that for large correlation (i. e. small antenna spacing), HS-MRC performs considerably worse than the others. Both FFTS and PSS approach the SNR gain of FC-MRC. With very low correlation, FFTS approaches the HS-MRC curve, while PSS has asymptotically a $4\mathrm{db}$ improvement in SNR gain.



(a) $\theta_r = \frac{\pi}{4}$, $L = 1$. (b) $\theta_r = \frac{\pi}{4}$, $L = 2$.
(c) $\theta_r = \frac{\pi}{6}$, $L = 1$. (d) $\theta_r = \frac{\pi}{6}$, $L = 2$.

Fig. 2. Performance comparison of the four schemes with the angle spread $\sigma_r = 6^o$ and antenna spacing $d = 0.5$ in a $t = 8$, $r = 8$ system.

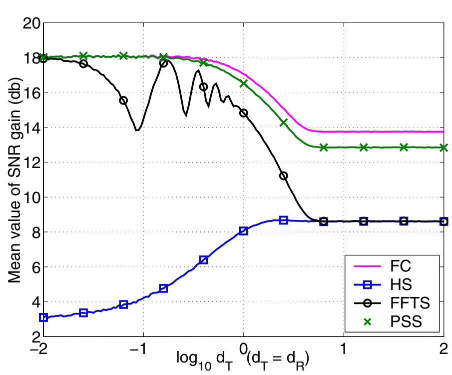


Fig. 3. SNR performance versus the antenna spacing with $\theta_r = \frac{\pi}{4}$, angle spread $\sigma_r = \frac{\pi}{30}$, $t = r = 8$ and $L = 1$.

VI. Conclusion

In this paper we presented a new antenna subset selection scheme in a multiple antenna system. Standard antenna selection, selecting L out of r antenna elements, and maximum-ratio combining them, exhibits a simple implementation with fewer demodulators, but loses the average signal power. This loss is largest in highly correlated channels, but can also be significant in uncorrelated fading environment. By embedding variable phase shifters in the RF chains before selection, our new system shows a significant advantage in improving the SNR gain with a much more reduced complexity for frequency conversion. Regardless of the correlation level in the channel, our approach can outperform both the traditional HS-MRC and FFT-based selection schemes. With two or more branches selected, the new design exhibits the full SNR gain of the original MIMO channel with full-complexity reception. The receiver side selection can be applied to the transmitter end in duality, which inspires the joint transceiver selection [10] under investigation.

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