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Keywords- Power, relay-assisted, wireless, regenerator

I. INTRODUCTION

Some of the target applications of wireless sensor networks are home and building automation, structural and environmental monitoring and intrusion detection etc. Depending on the nature of applications, wireless sensor networks can contain tens of thousands of sensing nodes operating on limited battery power. Therefore, it is desirable to minimize overall power consumption in the network, and accordingly to increase network lifetime. Some of the approaches to achieve power efficiency are to decrease encoder and decoder complexities, to design low power circuitries, and to develop low signaling-cost routing protocols etc. There are also other techniques that try to exploit network topology by promoting local collaboration among neighboring sensor nodes to reduce power expenditure. Relay assisted transmission and power combining methods with diversity techniques [1]-[4] fall under this category.

Assume a triplet as illustrated in Fig.1. In [1], [2] and [4], it is shown that when terminal A sends its data to B through C, the overall power consumption compared to the case when A transmits directly to B is reduced. This comparison is only valid provided that the two scenarios provision the same BER constraint, Pr_e^{ab}, at destination B. In [1-4], this constraint is provisioned. However, their results do not take transmit power compensation due to error propagation into account at the relay node. This extra power compensation would degrade power savings in simple relaying. On the other hand, the regeneration and forwarding process also consumes additional power in its circuitry. This is due to data manipulation such as decoding, error correction and re-encoding. The question to answer is "Is simple relaying with power compensation advantageous to regenerating and forwarding at the intermediate node from power efficiency point of view?"

In the regenerator mode (see Fig.1-top), the intermediate node C receives data from A, at a power level that would satisfy Pr_e^{ab} at the receiving end C, corrects bits in error and forwards the recovered data to B at a transmit power level that would again satisfy Pr_e^{ab} at receiver B. In a simple relay mode (see Fig.1-bottom), C does not perform any data recovery (e.g., FEC etc.), but cooperates with A such that A adjusts its transmit power level to provision Pr_e^{ac} at receiver C; and C adjusts its transmit power level to provision Pr_e^{cb} for the retransmitted data at receiver B. Hence, assuming that the bit errors on these two paths are additive in the simple relay case, the end-to-end BER is constrained to Pr_e^{ab} .

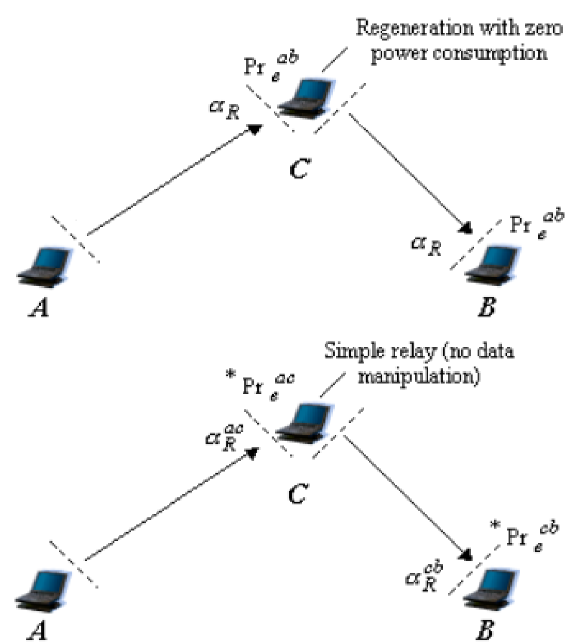

Figure 1 Illustration of the received signal power levels and bit error rates at receiving terminals in wireless transmission over (top) the regenerator (bottom) simple relay intermediate node.

II. RADIO MODEL

We denote by α_R the required received signal power at transmit rate *R* such that the BER at the receiver is $\Pr_e^{ab}$. We use a simple radio model analogous to the ones in [2], [3] to model wireless transmission. We assume that the power consumption of a communications node, P_t (in watts), consists of power consumed by the tx/rx radio circuitry, P_{circ}, which is fixed; the power consumed by the tx amplifier power, $P_{amp}(R)$, which is adjusted to achieve the desired BER at rate *R*; and power consumed to receive the bits at the receiver end, $P_{rec}(R)$. Also, different levels of clutter on propagation paths are modeled as *lognormal shadowing*. The shadowing effect between nodes *i* and *j*, x_{ij} (in dB), is expressed in the power equations as $10^{x_{ij}/10}$ (in watts).

$$P_t = P_{circ} + P_{amp}(R) + P_{rec}(R) \tag{1}$$

P_{amp} can be further expanded as $P_{amp}^{ij} = \alpha_R 10^{x_{ij}/10} d_{ij}^{\gamma}$, where d_{ij} is the distance between node pairs *i* and *j*, and γ is the path loss exponent. Antenna gain is assumed to be unity.

III. POWER ANALYSIS

In this section, we quantify power dissipation in a communications triplet with an intermediate node that is used as a regenerator and a simple-relay (amplifier). These two cases are compared to the power consumption in direct transmission.

A Direct Transmission

Let P_{amp}^{ij} denote the amplifier power at transmitter *i* for transmitting data to receiver *j*. When data are directly transmitted from *A* to *B*, the total power expended in the system, $P_t^{'}$, is

$$\begin{aligned} P_t^{'} &= P_{circ}^{a} + P_{amp}^{ab}(R) + P_{rec}(R) + P_{circ}^{b} \\ &= \quad (2P_c + P_{rec}(R)) + \alpha_R 10^{x_{ab}/10} d_{ab}^{\gamma} \end{aligned} \tag{2}$$

In (2), we have made an assumption without loss of generality that $P_{circ}^{a} = P_{circ}^{b} = P_{circ}^{c} = P_c$.

B. Intermediate Regenerator Node

Assume that A transmits data to *B* using *C* as an intermediate regenerator node. *C* would consume power while both receiving data from A and forwarding it to B. Let $P_t^{''}$ denote the power dissipation in this triplet.

$$\begin{aligned} P_t^{''} &= P_{circ}^{a} + P_{amp}^{ac}(R) + P_{circ}^{c} + 2P_{rec}(R) + P_{amp}^{cb}(R) + P_{circ}^{b} \\ &= 3P_c + 2P_{rec}(R) + \alpha_R (10^{x_{ac}/10} d_{ac}^{\gamma} + 10^{x_{cb}/10} d_{cb}^{\gamma}) \end{aligned} \tag{3}$$

The satisfactory condition to have power savings with comparison to the direct transmission is $\Delta P_1 = P_t^{''} - P_t^{'} < 0$.

$$\begin{aligned} \Delta P_1 &= \alpha_R (-10^{x_{ab}/10} d_{ab}^{\gamma} + 10^{x_{ac}/10} d_{ac}^{\gamma} \\ &+ 10^{x_{cb}/10} d_{cb}^{\gamma}) + P_c + P_{rec}(R) < 0 \end{aligned} \tag{4}$$

After rearranging (4), the test condition for power savings is found as in (5).

$$10^{x_{ac}/10} d_{ac}^{\gamma} + 10^{x_{cb}/10} d_{cb}^{\gamma} < 10^{x_{ab}/10} d_{ab}^{\gamma} - \frac{P_c + P_{rec}(R)}{\alpha_R} \tag{5}$$

Definition 1: The position of the intermediate node C that satisfies $\Delta P_1 = P_t^{''} - P_t^{'} < 0$ is called the *Feasible Relay Point* (FRP).

Definition 2: The set of all FRPs for a given source destination pair is called the *Feasible Relay Region* (FRR).

C. Intermediate Simple-Relay Node

In this scenario, C does not perform any recovery on the data received from A. Let α_R^{ac} and α_R^{cb} denote the required received signal power levels to provision $\Pr_e^{ac}$ from A to C and $\Pr_e^{cb}$ from C to B. Then, the overall power dissipation, $P_t^{'''}$, is

$$\begin{aligned} P_t^{'''} &= P_{circ}^{a} + P_{amp}^{ac}(R) + P_{circ}^{c} + 2P_{rec}(R) + P_{amp}^{cb}(R) + P_{circ}^{b} \\ &= 3P_c + 2P_{rec}(R) + \alpha_R^{ac} 10^{x_{ac}/10} d_{ac}^{\gamma} + \alpha_R^{cb} 10^{x_{cb}/10} d_{cb}^{\gamma} \end{aligned} \tag{6}$$

The relative power savings between simple relaying and direct transmission, $\Delta P_2 = P_t^{'''} - P_t^{'} < 0$, is given in (7).

$$\begin{aligned} &\Delta P_2 = \alpha_R^{ac} 10^{x_{ac}/10} d_{ac}^{\gamma} + \alpha_R^{cb} 10^{x_{cb}/10} d_{cb}^{\gamma} \\ &-\alpha_R 10^{x_{ab}/10} d_{ab}^{\gamma} + P_c + P_{rec}(R) < 0 \end{aligned} \tag{7}$$

Eq.7 leads to test condition (8), which is slightly different from (5).

$$\begin{aligned} &\frac{\alpha_R^{ac}}{\alpha_R} 10^{x_{ac}/10} d_{ac}^{\gamma} + \frac{\alpha_R^{cb}}{\alpha_R} 10^{x_{cb}/10} d_{cb}^{\gamma} \\ &< 10^{x_{ab}/10} d_{ab}^{\gamma} - \frac{P_c + P_{rec}(R)}{\alpha_R} \end{aligned} \tag{8}$$

Note that the right sides of (5) and (8) are the same. In order to be able to compare power consumption in the intermediate simple relay case to either the direct transmission or to the intermediate regenerator case, α_R^{cb} and α_R^{ac} must be selected such that the left side of (8) is minimized, under the constraint that $\Pr_e^{ac} + \Pr_e^{cb} = \Pr_e^{ab}$. Assuming a *M*-ary QAM modulation scheme in direct transmission from A to B, $\Pr_e^{ab}$ is approximated as in (9).

$$\Pr_e^{ab} = \frac{4(1 - 1/\sqrt{M})}{\log_2 M} Q\left(\sqrt{\frac{3\alpha_R \log_2 M}{R(M-1)N_0}}\right) \tag{9}$$

where N_0 is the AWGN noise power density. In a similar way, we can formulate $\Pr_e^{ac}$ and $\Pr_e^{cb}$ as in (10) and (11) respectively.

$$\mathrm{Pr}_e^{ac} = \frac{4(1-1/\sqrt{M})}{\log_2 M} Q\left(\sqrt{\frac{3\alpha_R^{ac}\log_2 M}{R(M-1)N_0}}\right) \tag{10}$$
$$= (4/\beta) Q\left(\sqrt{\alpha_R^{ac}/\alpha_R}\, Q^{-1}\left(0.25\beta \mathrm{Pr}_e^{ab}\right)\right)$$

$$\mathrm{Pr}_e^{cb} = \frac{4(1-1/\sqrt{M})}{\log_2 M} Q\left(\sqrt{\frac{3\alpha_R^{cb}\log_2 M}{R(M-1)N_0}}\right) \tag{11}$$
$$= (4/\beta) Q\left(\sqrt{\alpha_R^{cb}/\alpha_R}\, Q^{-1}\left(0.25\beta \mathrm{Pr}_e^{ab}\right)\right)$$

where $\beta = (\log_2 M)/(1-1/\sqrt{M})$.

The goal is to find the optimum error rates $^*\mathrm{Pr}_e^{ac}$ and $^*\mathrm{Pr}_e^{cb}$ that would minimize objective function (12), and then to compute the optimum received signal power levels $^*\alpha_R^{ac}$ and $^*\alpha_R^{cb}$.

$$\arg\min_{\mathrm{Pr}_e^{ac}} \left\{ \begin{array}{l} \left(\dfrac{Q^{-1}(0.25\beta \mathrm{Pr}_e^{ac})}{Q^{-1}(0.25\beta \mathrm{Pr}_e^{ab})}\right)^2 10^{x_{ac}/10} d_{ac}^{\gamma} \\ + \left(\dfrac{Q^{-1}(0.25\beta \mathrm{Pr}_e^{cb})}{Q^{-1}(0.25\beta \mathrm{Pr}_e^{ab})}\right)^2 10^{x_{cb}/10} d_{cb}^{\gamma} \end{array} \right\} \tag{12}$$

After rearranging the derivative of (12) in terms of Pr_e^{ac}, we get (13). It is then straightforward to find $^*\mathrm{Pr}_e^{ac}$.

$$\ln\left(\frac{d_{ac}^{\gamma} 10^{x_{ac}/10}}{d_{cb}^{\gamma} 10^{x_{cb}/10}}\right) = \ln\left(Q^{-1}\left(0.25\beta(\mathrm{Pr}_e^{ab} - \mathrm{Pr}_e^{ac})\right)\right)$$
$$+ \left(erf^{-1}\left(1 - 0.5\beta(\mathrm{Pr}_e^{ab} - \mathrm{Pr}_e^{ac})\right)\right)^2 - \left(erf^{-1}\left(1 - 0.5\beta \mathrm{Pr}_e^{ac}\right)\right)^2 \tag{13}$$
$$- \ln\left(Q^{-1}\left(0.25\beta \mathrm{Pr}_e^{ac}\right)\right)$$

Note that $^*\mathrm{Pr}_e^{cb} = \mathrm{Pr}_e^{ab} - {}^*\mathrm{Pr}_e^{ac}$. Finally, $^*\alpha_R^{ac}$ and $^*\alpha_R^{cb}$ are derived from (10) and (11), and are used in (6) to compute power dissipation in the simple relay triplet.

IV. Lognormal Shadowing Analysis

In the previous analysis, the left sides of inequalities (5) and (8) are in the form of a sum of two lognormal variables with different means and variances. For some environments, it is acceptable to assume that the shadowing on each communication path is uncorrelated or the correlation is negligible. We leave the analysis of correlated shadowing to our next study, and (for the current work) assume uncorrelated shadowing on communication paths of the triplet. In [5], they derive upper and lower bounds on the distribution function of the sum of independent lognormal variables. We exploit the results in [5] to express the lower (14) and upper bounds (15) for the probability (for both simple relaying and regenerating cases) that relaying is more power efficient than the direct transmission at a given intermediate relay node position.

$$F_L(x) = 1 - \prod_{k=1}^{2}[1 - Q(\frac{\ln(x/2) - m_{x_k}}{\sigma_{x_k}})] \tag{14}$$

$$F_U(x) = 1 - \prod_{k=1}^{2}[1 - Q(\frac{\ln(x) - m_{x_k}}{\sigma_{x_k}})] \tag{15}$$

The function *Q(.)* represents the complementary cdf of a zero mean, unit variance Gaussian random variable. The definition of each variable in (14) and (15) are as follows, considering $I_1 = (\alpha_R^{ac}/\alpha_R)10^{x_{ac}/10} d_{ac}^{\gamma}$ and $I_2 = (\alpha_R^{cb}/\alpha_R)10^{x_{cb}/10} d_{cb}^{\gamma}$.

x: The threshold in (5) and (8) to achieve power savings in relaying with respect to direct transmission. Note that $x_{ab} \to N(0,\sigma)$.

m_{x_1}, m_{x_2}: The expected values of random variables I_1 and I_2 respectively.

σ_{x_1}, σ_{x_1}: Standard deviations of the random variables I_1 and I_2 respectively. We assume $\sigma_{x_1} = \sigma_{x_2} = \sigma$.

Please note that in the regenerator scenario $\alpha_R^{ac}/\alpha_R = 1$.

V. Results

In this section, we first would like to quantify how much the upper and lower bounds given in (14) and (15) differ in simple relay and regenerator intermediate node scenarios. Figure 2 shows the probability of "an intermediate node position $C(d_{ac}, y)$ on the direct path (y=0) to be a FRP. The probability of this is expressed as

$P(C(d_{ac}, y) \in FRP \mid y = 0 \cap d_{ac} = 1,2...,99)$.

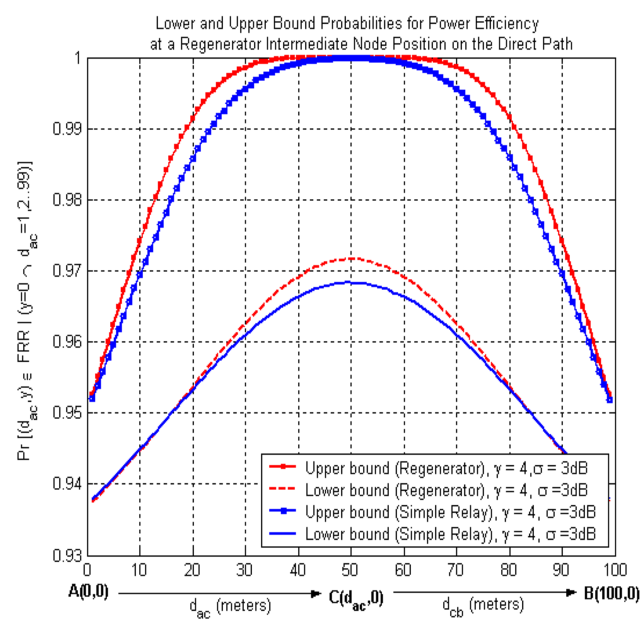

Figure 2 Illustration of the probability of an intermediate node position on the direct path to be a FRP. (Note: $\mathrm{Pr}_e^{ab} = 10^{-3}$, γ=4, σ = 3, M=16)

It is clear that both upper and lower bounds for the regenerator case has higher probabilities than the simple relay, the maximum probability is achieved at the midpoint. In other words, a given intermediate node position is more likely to be a FRP, if the intermediate node performs regenerating and forwarding rather than simple relaying. However, we should point out the fact that this comparison does not take into account the extra power depletion due to regenerating process, which would narrow the margin between performance curves of the regenerator and simple relay in Fig.2.

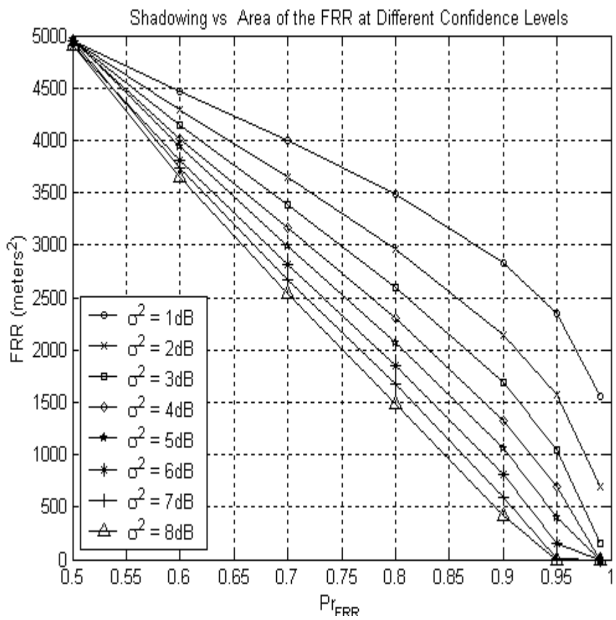

Figure 3 Illustration of the area of FRR vs. the level of confidence in the regenerator case under different levels of shadowing effect. Note: $\mathrm{Pr}_e^{ab} = 10^{-3}$, γ=4, $(P_c + P_{rec}(R))/\alpha_R = 0.1$.

Second, we compute the area of the FRR for different values of path loss exponent and the variance of lognormal shadowing for the regenerator case after 1E6 independent realizations, since it is shown to be outperforming the simple relay as illustrated in Fig.2. This result is given in Fig.3. It is clear that the size of the FRR is inversely proportional to the variance of the lognormal shadowing σ^2 on each path. In other words, at a required confidence level, the size of the FRR gets smaller as σ^2 increases.

The size of the FRR at confidence level c (where $0<c<1$) is computed by counting the number of intermediate node positions that would be a FRP at least $c10^6$ times after 1E6 realizations as illustrated in Fig.4.

Third, given a *100m* by *100m-square* region, we determine the positions of the intermediate node at which regeneration would save more power than simple relaying. The Cartesian

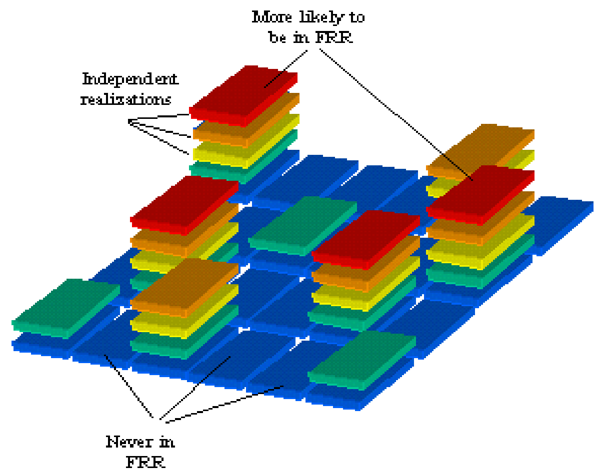

Figure 4 Illustration of the method of computing confidence levels for the size of FRR.

coordinates of *A, B* and *C* are denoted by, $(0,0)$, $(100,0)$ and (i,j) respectively, $i,j\in[1,99]$. We define a metric that is the average of the ratio of the power expenditure of the ABC triplet in the relay mode to that in the regenerator mode over every intermediate node position satisfying $P_t^{'} > P_t^{'''}$ (16).

$$E[\frac{P_t^{'''}}{P_t^{''}}] = \frac{\sum_{i=1}^{99}\sum_{j=1}^{99}\{P_t^{'''}(i,j)/P_t^{''}(i,j) \mid P_t^{'''}(i,j) < P^{'}\}}{\sum_{i=1}^{99}\sum_{j=1}^{99}\{1 \mid P_t^{'''}(i,j) < P^{'}\}} \tag{16}$$

Figure 5 plots the numerator in (16) for every intermediate node position over the direct path between A and B. The results show that regenerating may expend up to 0.55dB less power compared to simple relaying, as the intermediate node gets closer to the midpoint of A-B separation.

The numerical results given in Table-I for (16) prove that for lower Pr_e^{ab} and higher γ, the performance margin between the simple relay and the regenerator gets quite narrow. Noting that results in Fig.5 and Table-I ignore power consumption due to regeneration process at the intermediate node, simple relaying may be still preferred to regenerating, if regenerating would dissipate more power than the margin given in Fig.5 at a specific intermediate node position.

TABLE 1

AVERAGE POWER SAVINGS DIFFERENCE BETWEEN SIMPLE-RELAY AND REGENERATOR MODES (IN dB)

γ		3	4	5	6
$E[\frac{P_t^{'''}}{P_t^{''}}]$	$\mathrm{Pr}_e^{ab} = 10^{-4}$	0.22	0.19	0.17	0.15
	$\mathrm{Pr}_e^{ab} = 10^{-3}$	0.31	0.27	0.23	0.21

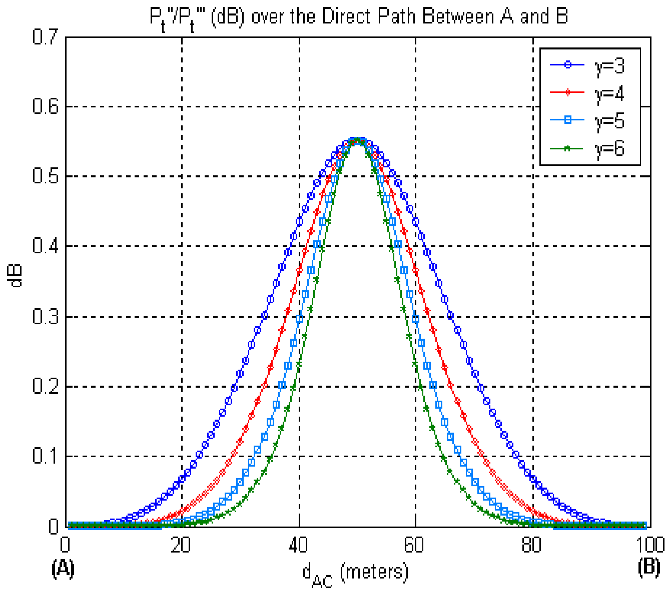

Figure 5 Power savings of regenerating with respect to simple relaying at intermediate node positions over the direct path between A and B separated with 100m.

Note: $P_{re}^{ab} = 10^{-3}$, $x_{ab}, x_{cb}, x_{ac} = 0$, M=64, and $(3P_c + 2P_{rec}(R))/\alpha_R = 0.2$.

In image sensor networks, where the transmitted bit stream is of an image or sequence of images, regeneration process may not be energy efficient, because the encoder and decoder computational complexities drastically increase the power consumption. It may easily exceed the 0.55dB margin for the specific scenario given in Fig.5. Please note that processing power consumption may even be dominant compared to the transmission cost. Therefore, simply amplifying and forwarding the bit stream signal may be preferred.

CONCLUSIONS

We quantify power savings performances of simple relaying and regenerating at an intermediate node in relay-assisted wireless transmission. In the simple relay case, we optimize transmit powers to compensate for error propagation. We also show how lognormal shadowing impacts the size of the FRR. Our future work includes studying the impacts of correlated shadowing and Raleigh fading on power savings in regenerator and simple-relay -assisted transmission.

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